

# Set 2: State-spaces and Uninformed Search

ICS 271 Fall 2018

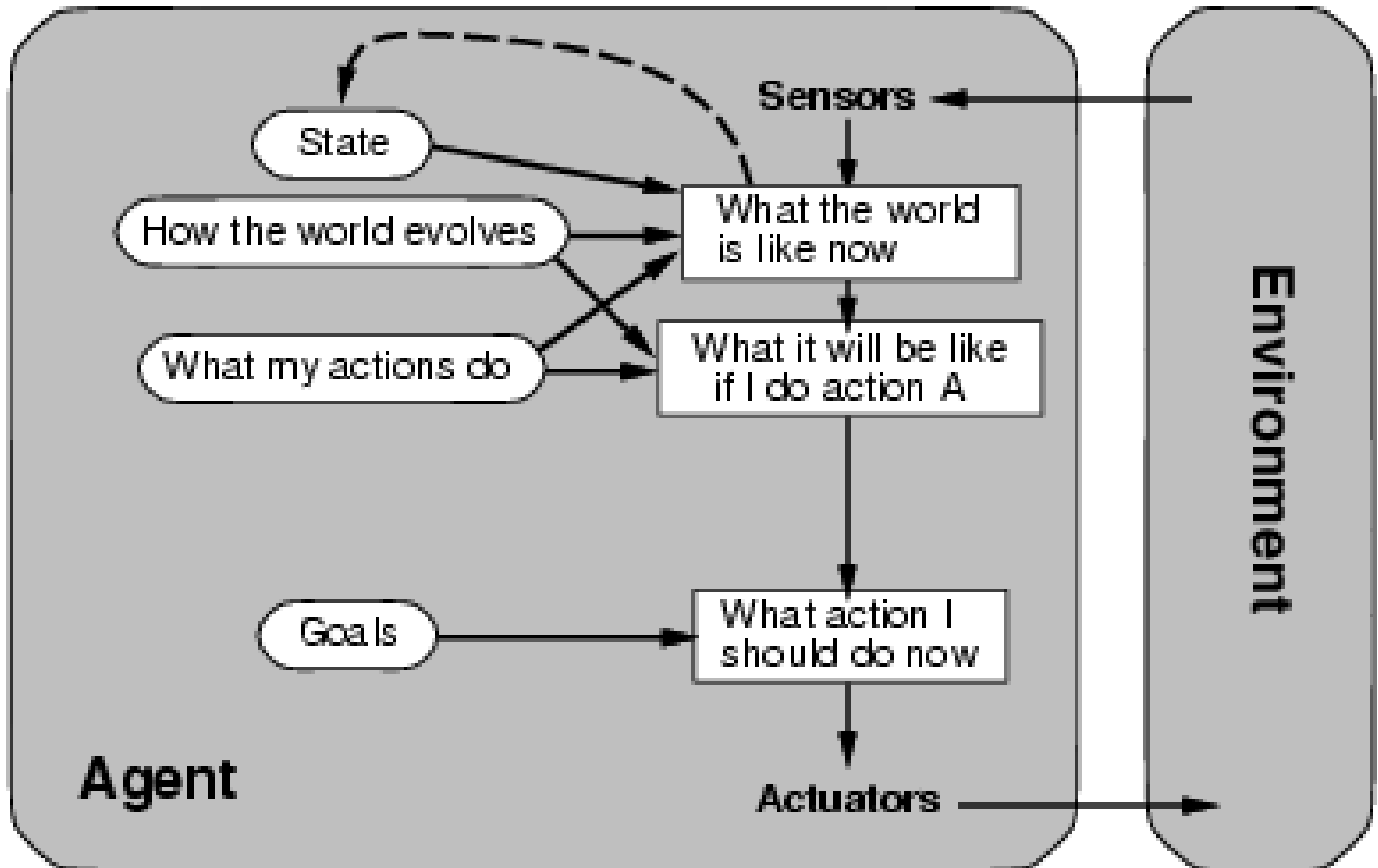
Kalev Kask

# You need to know

- State-space based problem formulation
  - State space (graph)
- Search space
  - Nodes vs. states
  - Tree search vs graph search
- Search strategies
- Analysis of search algorithms
  - Completeness, optimality, complexity
  - $b$ ,  $d$ ,  $m$

# Goal-based agents

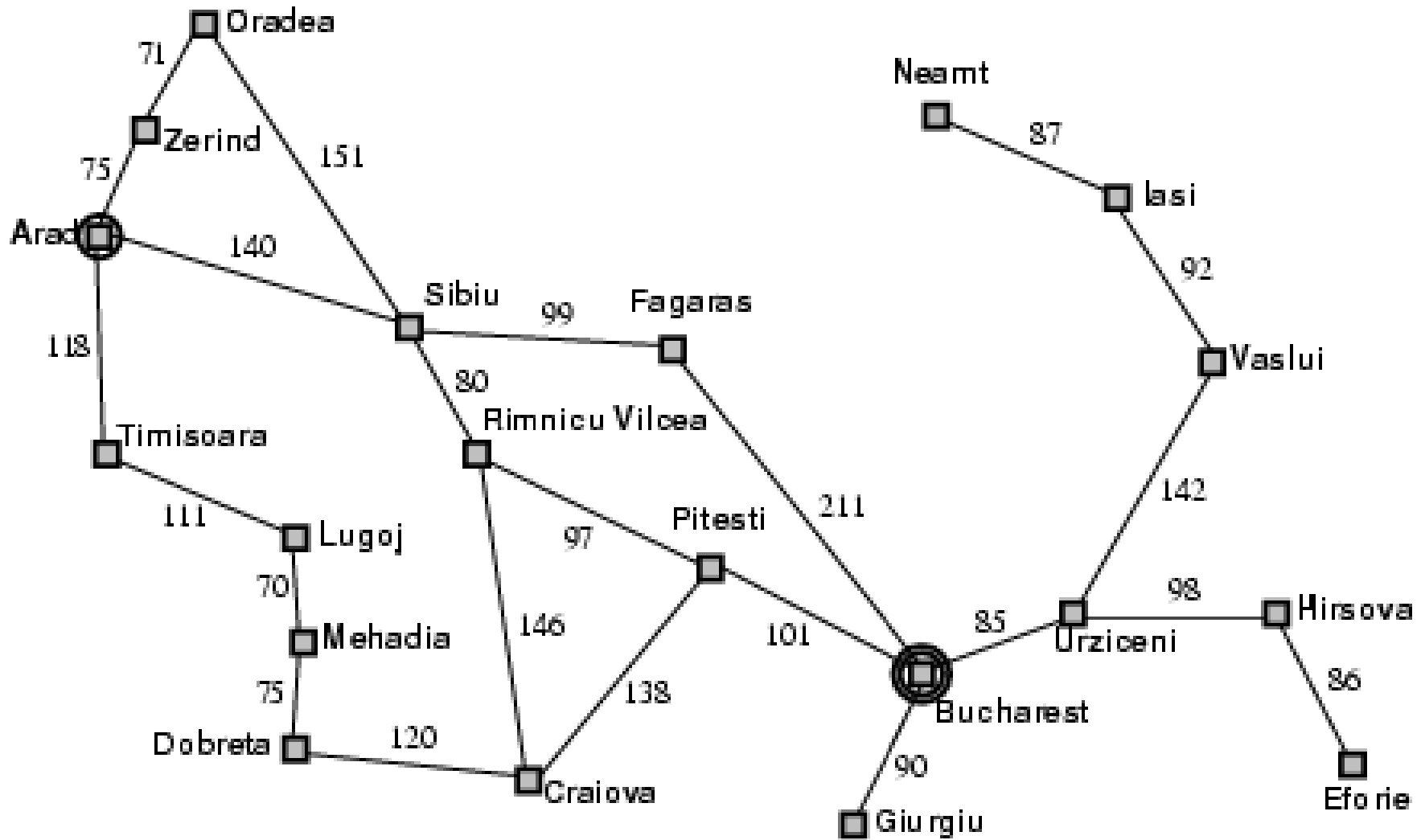
Goals provide reason to prefer one action over the other.  
We need to predict the future: we need to plan & search



# Problem-Solving Agents

- Intelligent agents can solve problems by searching a state-space
- State-space Model
  - the agent's model of the world
  - usually a set of discrete states
  - e.g., in driving, the states in the model could be towns/cities
- Goal State(s)
  - a goal is defined as a desirable state for an agent
  - there may be many states which satisfy the goal
    - e.g., drive to a town with a ski-resort
  - or just one state which satisfies the goal
    - e.g., drive to Mammoth
- Operators(actions)
  - operators are legal actions which the agent can take to move from one state to another

# Example: Romania



# Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- **Formulate goal:**
  - be in Bucharest
- **Formulate problem:**
  - **states:** various cities
  - **actions:** drive between cities
- **Find solution:**
  - sequence of actions (cities), e.g., Arad, Sibiu, Fagaras, Bucharest

# Environment Types

- **Static / Dynamic**

Previous problem was static: no attention to changes in environment

- **Observable / Partially Observable / Unobservable**

Previous problem was observable: it knew its initial state.

- **Deterministic / Stochastic**

Previous problem was deterministic: no new percepts were necessary, we can predict the future perfectly

- **Discrete / continuous**

Previous problem was discrete: we can enumerate all possibilities

# State-Space Problem Formulation

A **problem** is defined by five items:

**states** e.g. cities

**initial state** e.g., "at Arad"

**actions** or **successor function**  $S(x)$  = set of action–state pairs

- e.g.,  $S(\text{Arad}) = \{\langle \text{Arad} \rightarrow \text{Zerind}, \text{Zerind} \rangle, \dots \}$

**transition function** - maps *action & state*  $\rightarrow$  *state*

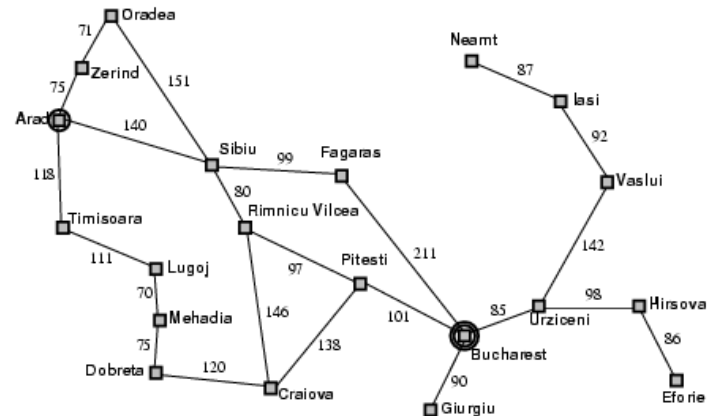
**goal test**, (or goal state)

e.g.,  $x = \text{"at Bucharest"}$ ,  $\text{Checkmate}(x)$

**path cost** (additive)

- e.g., sum of distances, number of actions executed, etc.
- $c(x,a,y)$  is the **step cost**, assumed to be  $\geq 0$

A **solution** is a sequence of actions leading from the initial state to a goal state





# State-Space Problem Formulation

- **A statement of a Search problem has components**
  - 1. States
  - 2. A start state  $S$
  - 3. A set of operators/actions which allow one to get from one state to another
  - 4. transition function
  - 5. A set of possible goal states  $G$ , or ways to test for goal states
  - 6. Cost path
- **A solution consists of**
  - a sequence of operators which transform  $S$  into a goal state  $G$
- **Representing real problems in a State-Space search framework**
  - may be many ways to represent states and operators
  - key idea: represent only the relevant aspects of the problem (**abstraction**)

# Abstraction/Modeling

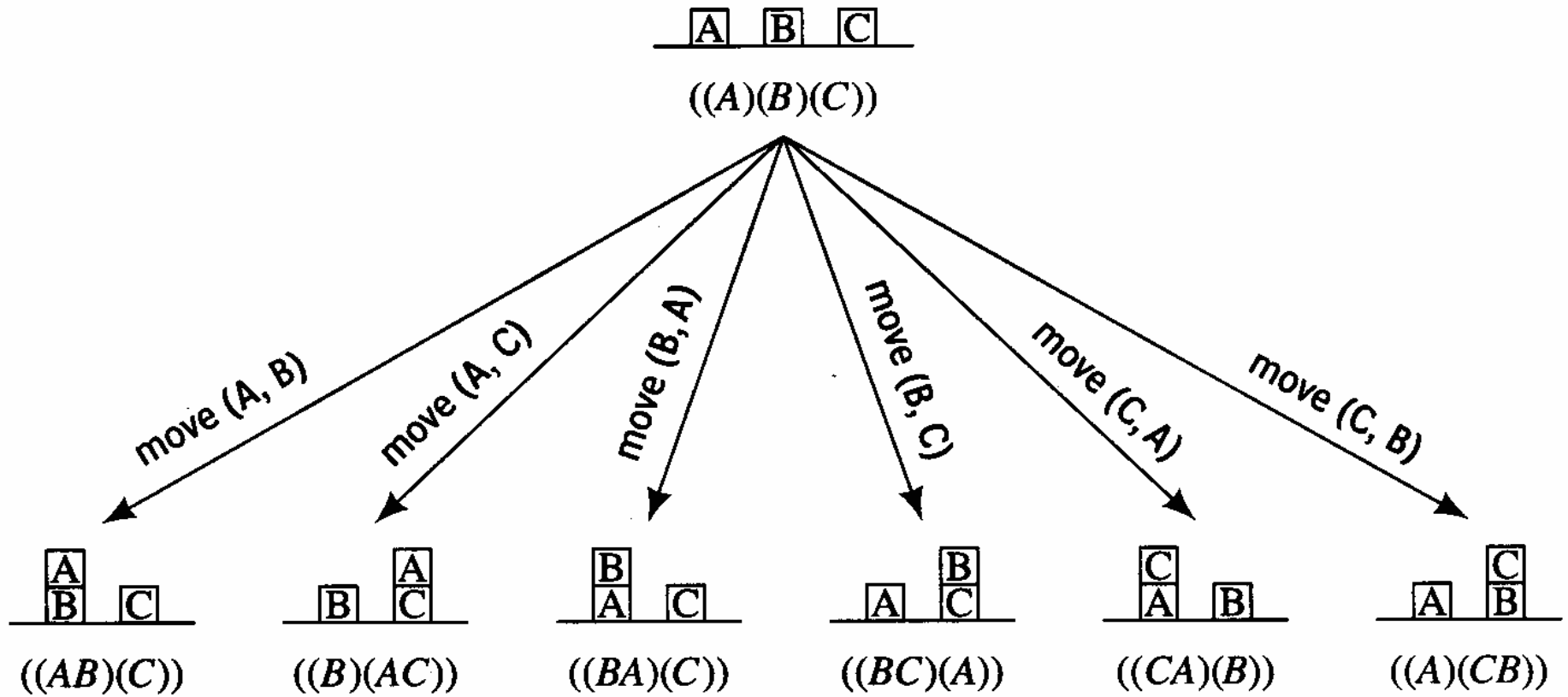
- **Definition of Abstraction (states/actions)**
  - Process of removing irrelevant detail to create an abstract representation: “high-level”, ignores irrelevant details
- **Navigation Example: how do we define states and operators?**
  - First step is to abstract “the big picture”
    - i.e., solve a map problem
    - nodes = cities, links = freeways/roads (a high-level description)
    - this description is an abstraction of the real problem
  - Can later worry about details like freeway onramps, refueling, etc
- **Abstraction is critical for automated problem solving**
  - must create an approximate, simplified, model of the world for the computer to deal with: real-world is too detailed to model exactly
  - good abstractions retain all important details
  - an abstraction should be easier to solve than the original problem

# Robot block world

- Given a set of blocks in a certain configuration,
- Move the blocks into a goal configuration.
- Example :
  - ((A)(B)(C))  $\rightarrow$  (ACB)



# Operator Description



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## Effects of Moving a Block

# The State-Space Graph

- **Problem formulation:**

- Give an abstract description of states, operators, initial state and goal state.

- **Graphs:**

- vertices, edges(arcs), directed arcs, paths

- **State-space graphs:**

- States are vertices
- operators are directed arcs
- solution is a path from start to goal

- **Problem solving activity:**

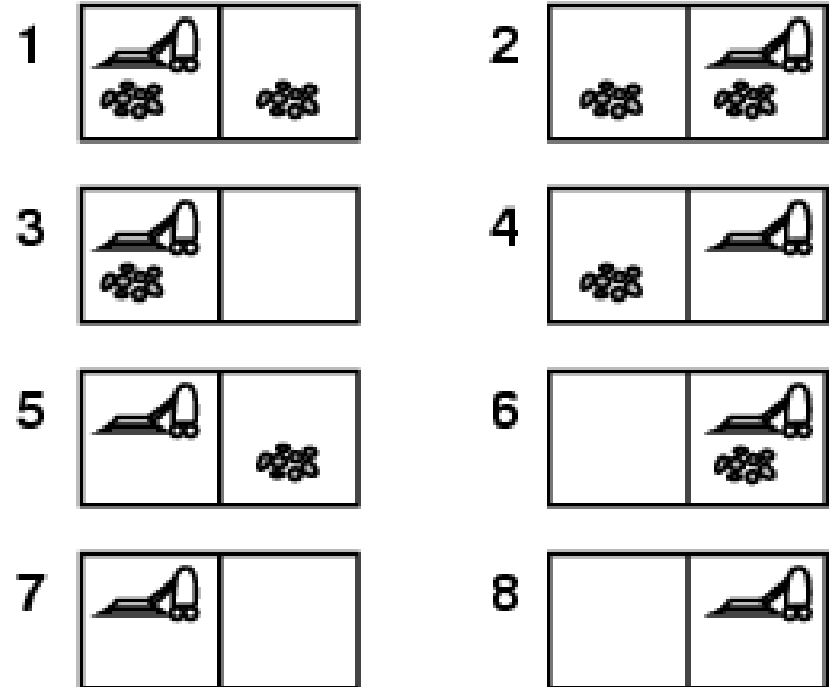
- Generate a part of the search space that contains a solution

**State-space:**

1. A set of states
2. A set of “operators”/transitions
3. A start state  $S$
4. A set of possible goal states
5. Cost path

# Example: vacuum world

- **Observable**, start in #5.  
Solution?

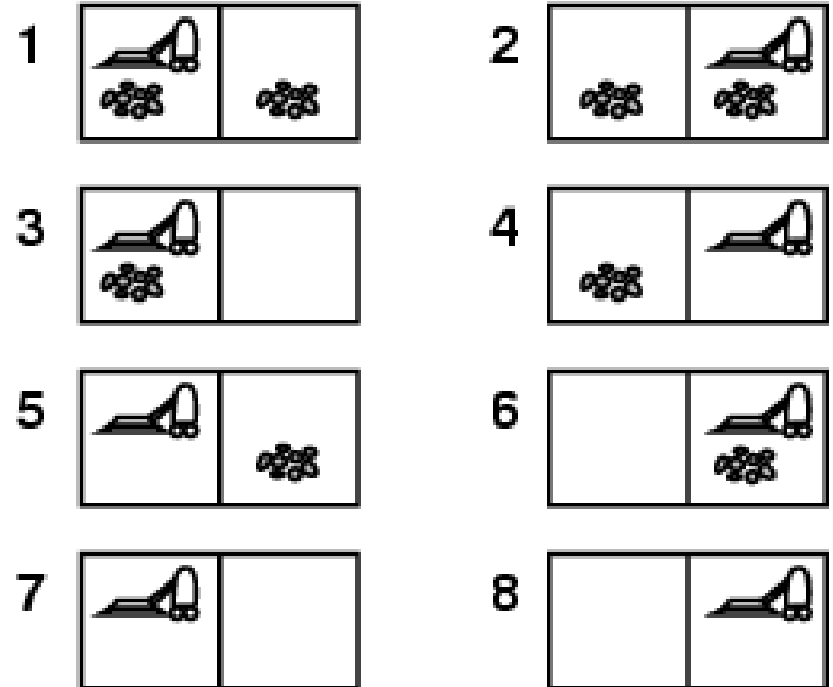


# Example: vacuum world

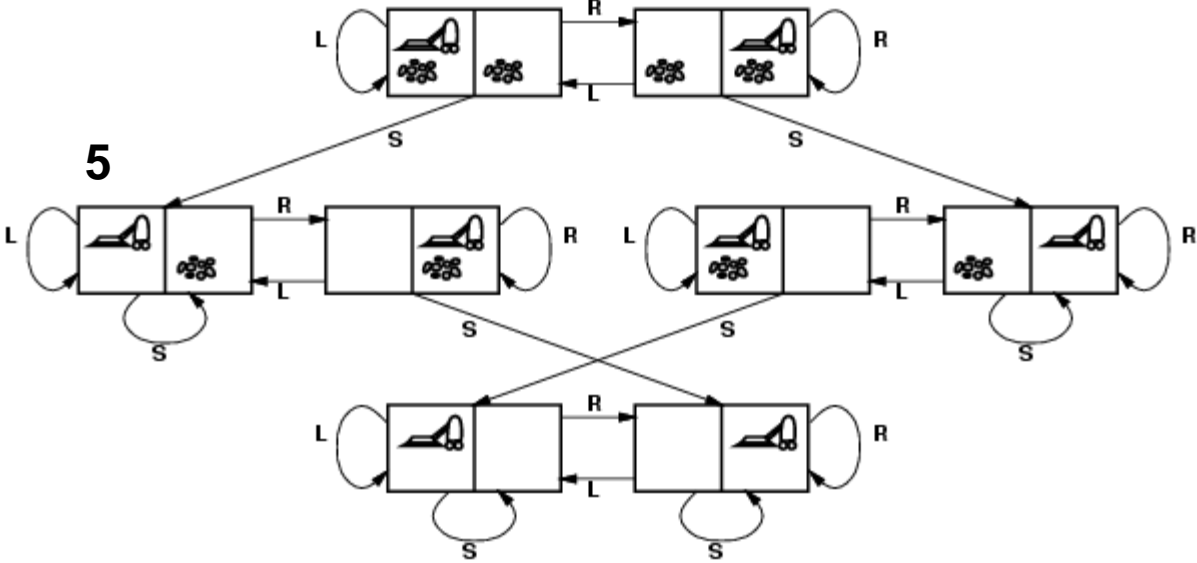
- **Observable**, start in #5.

Solution?

*[Right, Suck]*



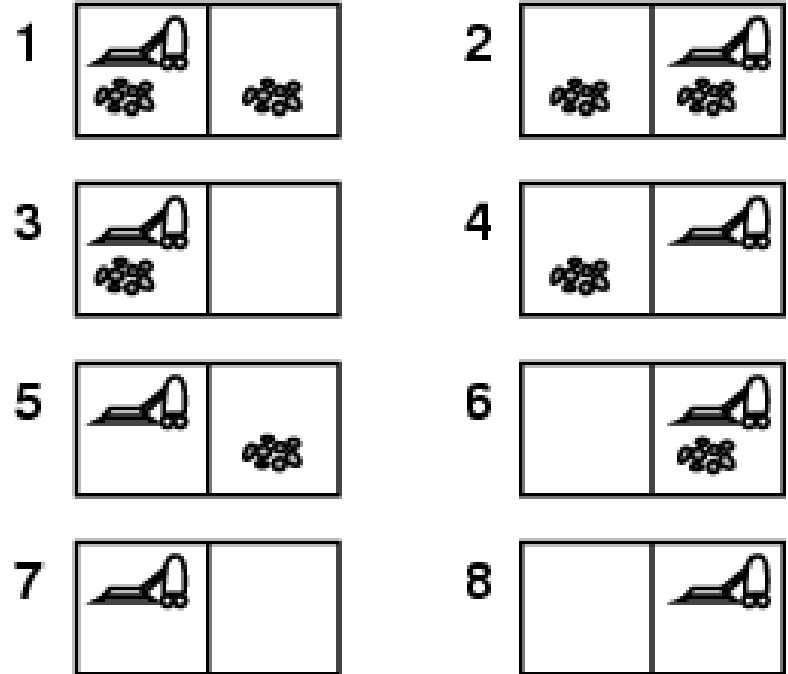
# Vacuum world state space graph





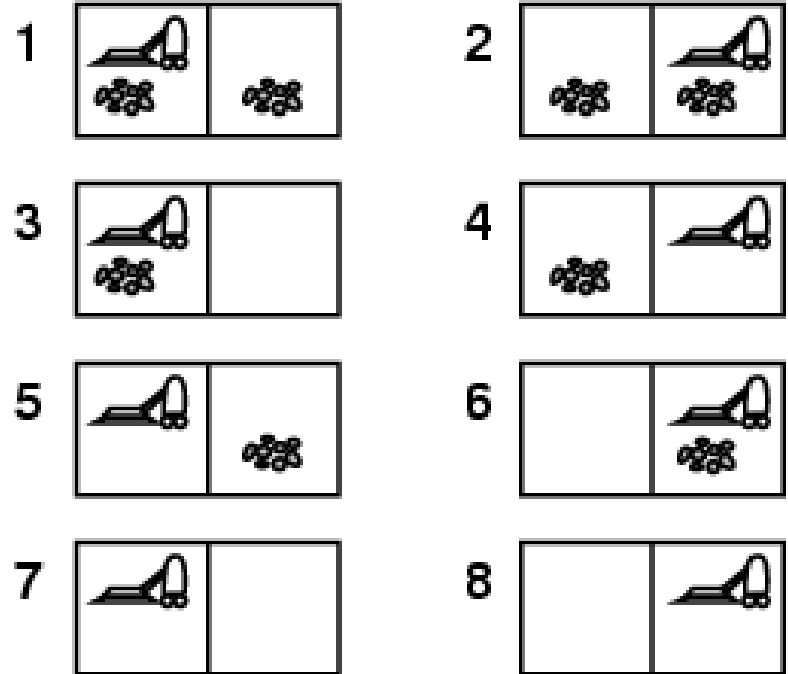
# Example: vacuum world

- Unobservable, start in {1,2,3,4,5,6,7,8} e.g., Solution?



# Example: vacuum world

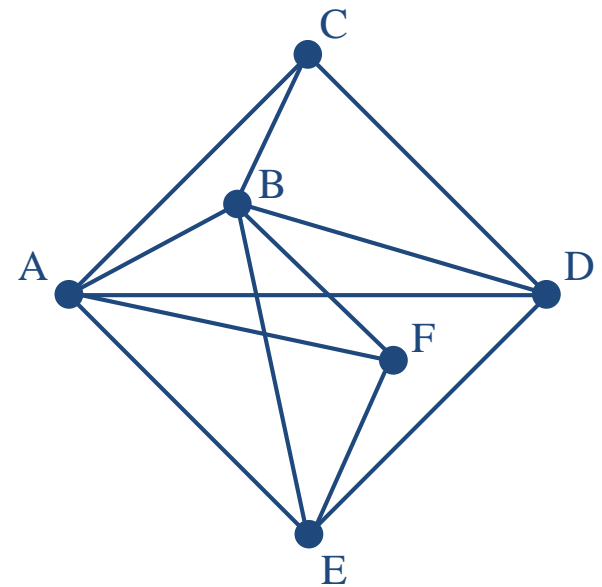
- Unobservable, start in {1,2,3,4,5,6,7,8} e.g.,  
Solution?  
*[Right,Suck,Left,Suck]*





# The Traveling Salesperson Problem

- Find the shortest tour that visits all cities without visiting any city twice and return to starting point.
- State:
  - sequence of cities visited
- $S_0 = A$

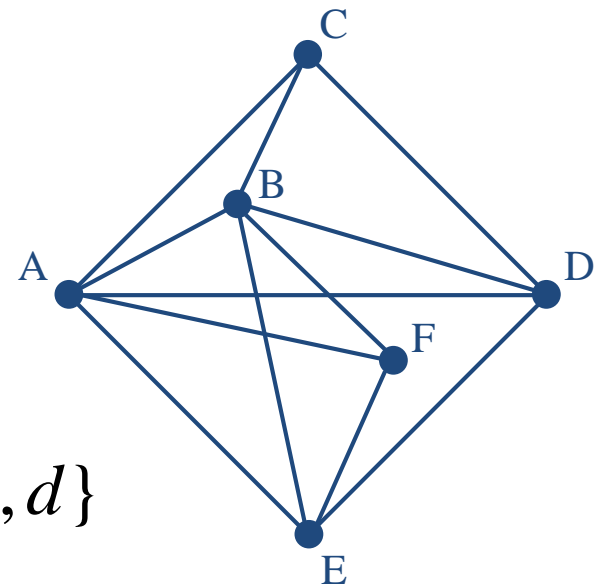


# The Traveling Salesperson Problem

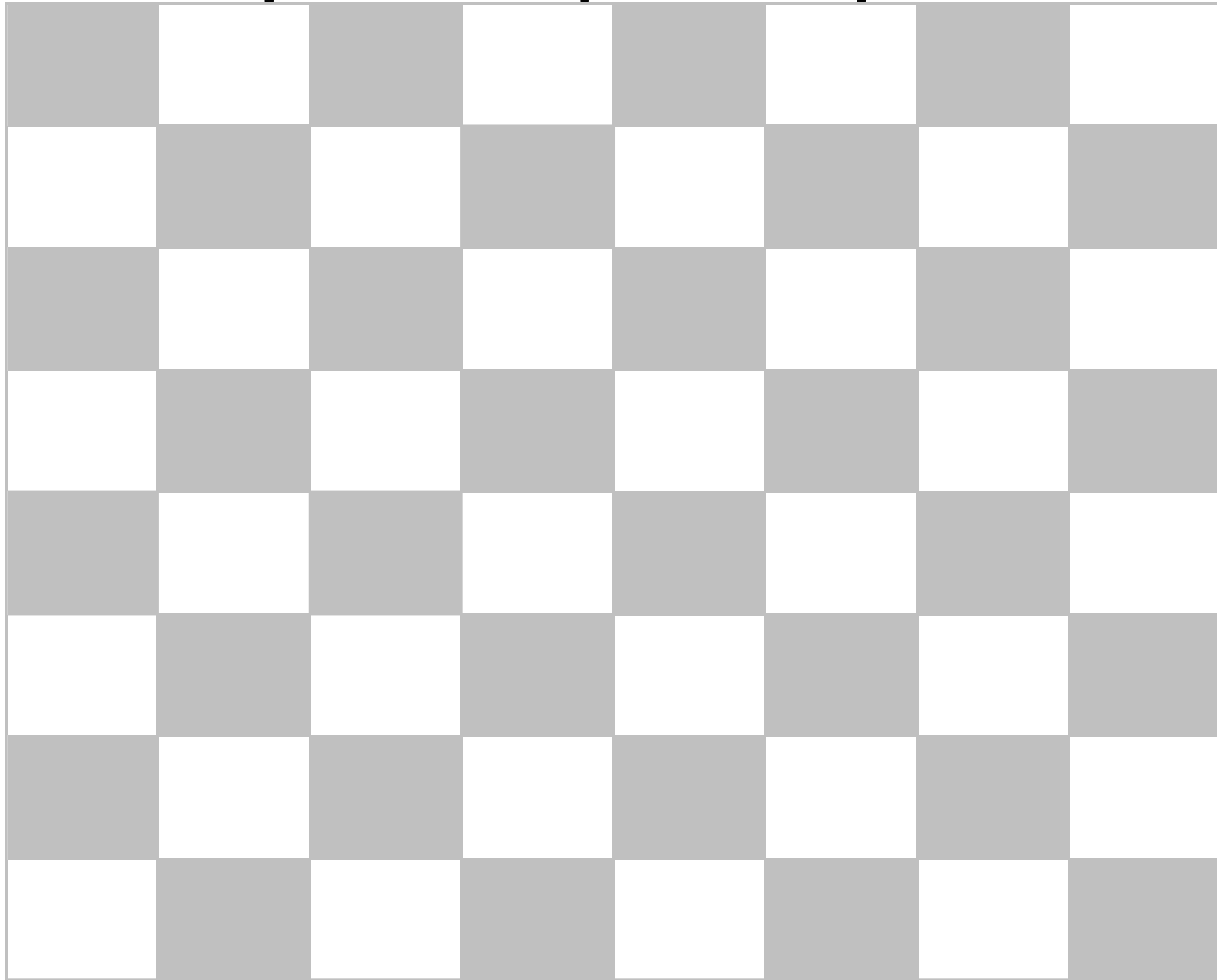
- Find the shortest tour that visits all cities without visiting any city twice and return to starting point.
- State: sequence of cities visited
- $S_0 = A$
- Solution = a complete tour

Transition model

$$\{a, c, d\} \Rightarrow \{(a, c, d, x) \mid X \notin a, c, d\}$$

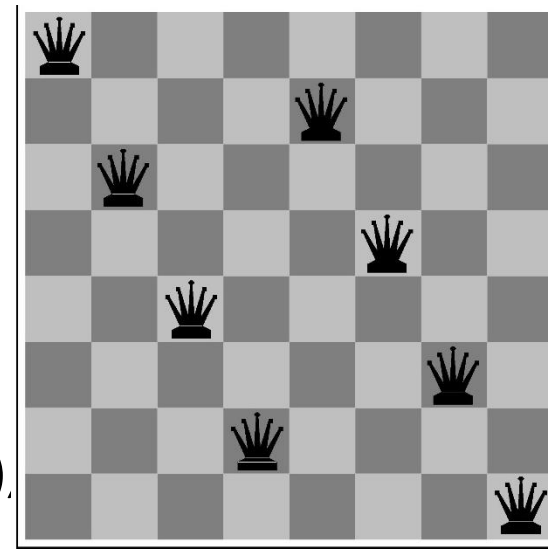


# Example: 8-queen problem



# Example: 8-Queens

- states? -any arrangement of  $n \leq 8$  queens
  - or arrangements of  $n \leq 8$  queens, 1 per column, such that no queen attacks any other (BETTER).
  - or arrangements of  $n \leq 8$  queens in leftmost  $n$  columns, 1 per column, such that no queen attacks any other (BEST)
- initial state? no queens on the board
- actions? -add queen to any empty column
  - or add queen to leftmost empty column such that it is not attacked by other queens.
- goal test? 8 queens on the board, none attacked.
- path cost? 1 per move



# The Sliding Tile Problem



**Figure 8.1**

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Start and Goal Configurations for the Eight-Puzzle

*move(x, loc y, loc z)*

Up  
Down  
Left  
Right



# The “8-Puzzle” Problem

**Start State**

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | ■ | 6 |
| 7 | 5 | 8 |



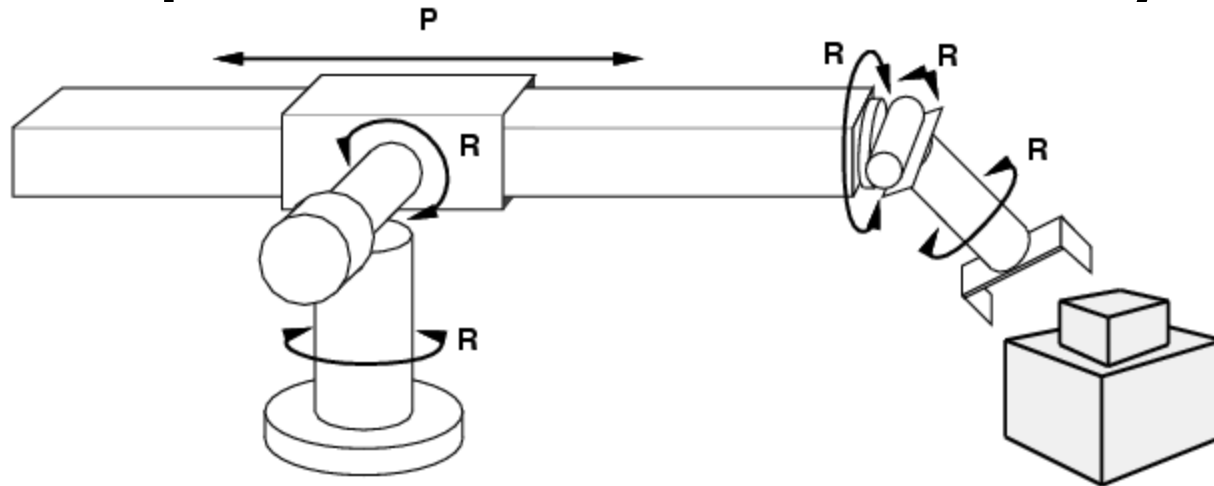
|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | ■ | 8 |



|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | ■ |

**Goal State**

# Example: robotic assembly



- states?: real-valued coordinates of robot joint angles  
parts of the object to be assembled
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

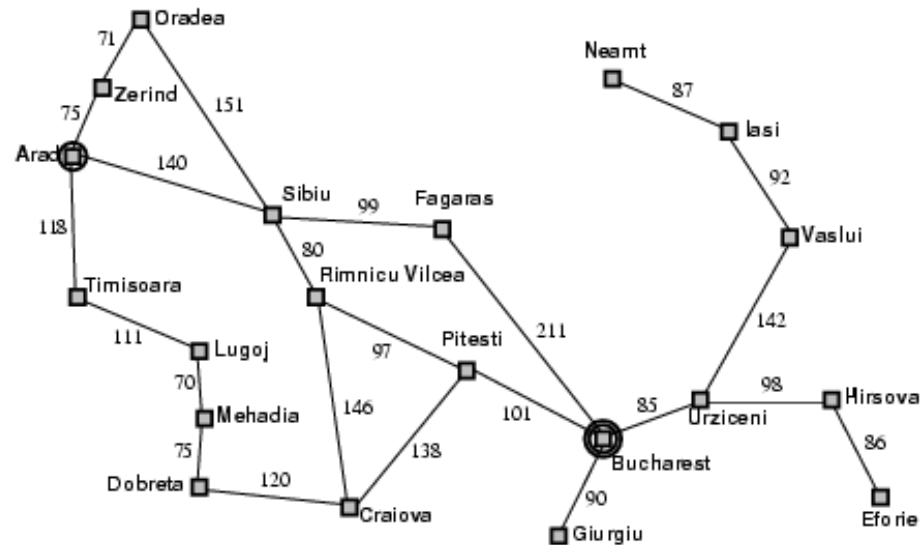
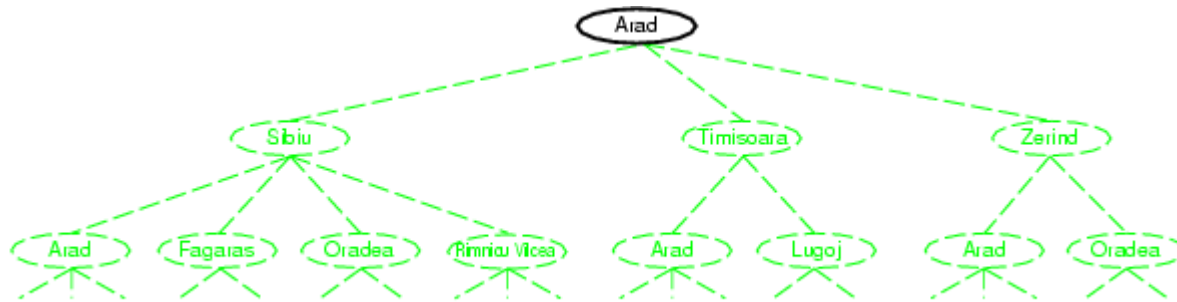
# Formulating Problems; Another Angle

- **Problem types**
  - Satisfying: 8-queen
  - Optimizing: Traveling salesperson
    - For traveling salesperson satisfying easy, optimizing hard
- **Goal types**
  - board configuration
  - sequence of moves
  - A strategy (contingency plan)
- **Satisfying leads to optimizing since “small is quick”**
- For traveling salesperson
  - satisfying easy, optimizing hard
- **Semi-optimizing:**
  - Find a good solution
- **In Russel and Norvig:**
  - single-state, multiple states, contingency plans, exploration problems

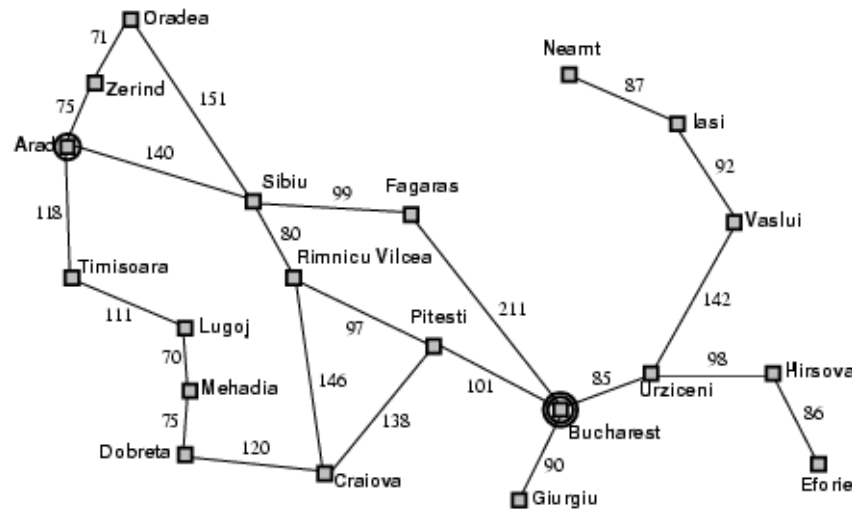
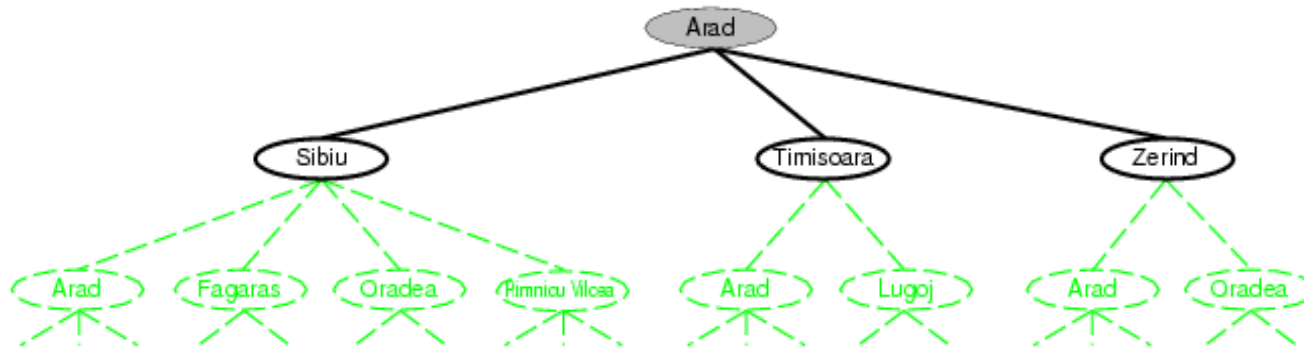
# Searching the State Space

- Exploration of the state space
  - states, operators
  - by generating successors of already explored states (aka **expanding** states)
- Trial and error : pick on possible extension of some path, leaving others aside for the time being.
- **Control strategy** (how to pick a node to expand) generates a search tree.
- Systematic search
  - Do not leave any stone unturned
- Efficiency
  - Do not turn any stone more than once

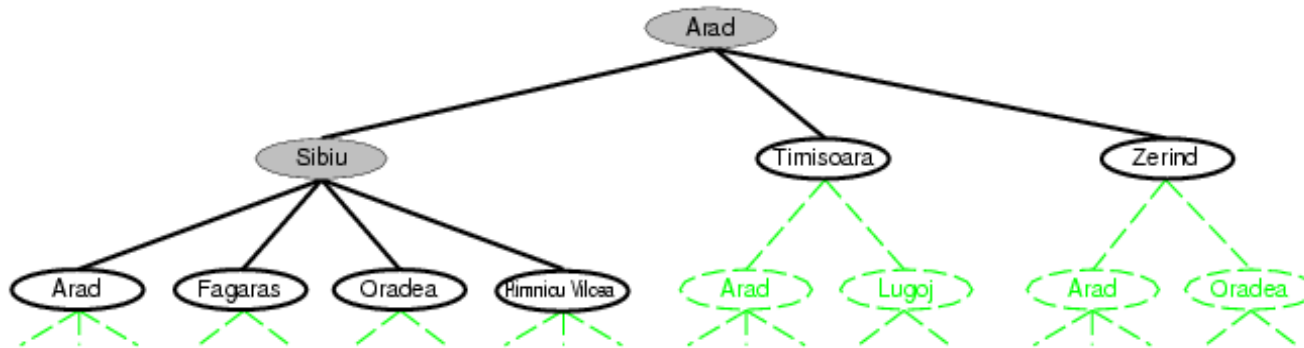
# Tree search example



# Tree search example

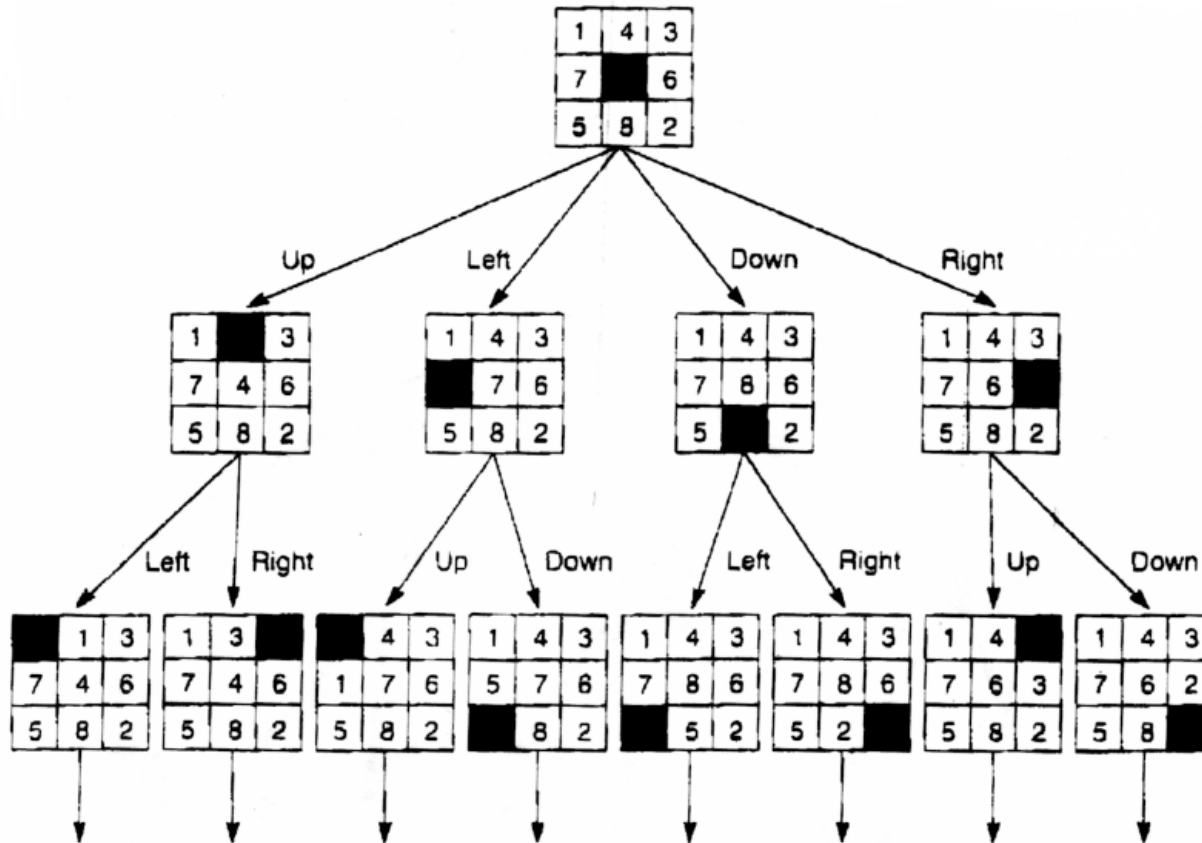


# Tree search example



**function** TREE-SEARCH(*problem, strategy*) **returns** a solution, or failure  
initialize the search tree using the initial state of *problem*  
**loop do**  
    **if** there are no candidates for expansion **then return** failure  
    choose a leaf node for expansion according to *strategy*  
    **if** the node contains a goal state **then return** the corresponding solution  
    **else** expand the node and add the resulting nodes to the search tree

# State-Space Graph of the 8 Puzzle Problem

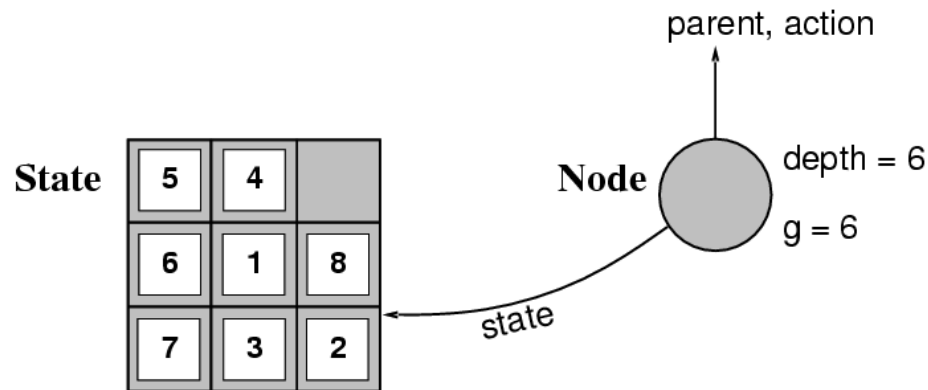


**Figure 3.6** State space of the 8-puzzle generated by "move blank" operations.



# Implementation

- States vs Nodes
  - A **state** is a (representation of) a physical configuration
  - A **node** is a data structure constituting part of a search tree contains info such as: **state**, **parent node**, **action**, **path cost  $g(x)$** , **depth**



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.
- Queue managing frontier :
  - FIFO
  - LIFO
  - priority

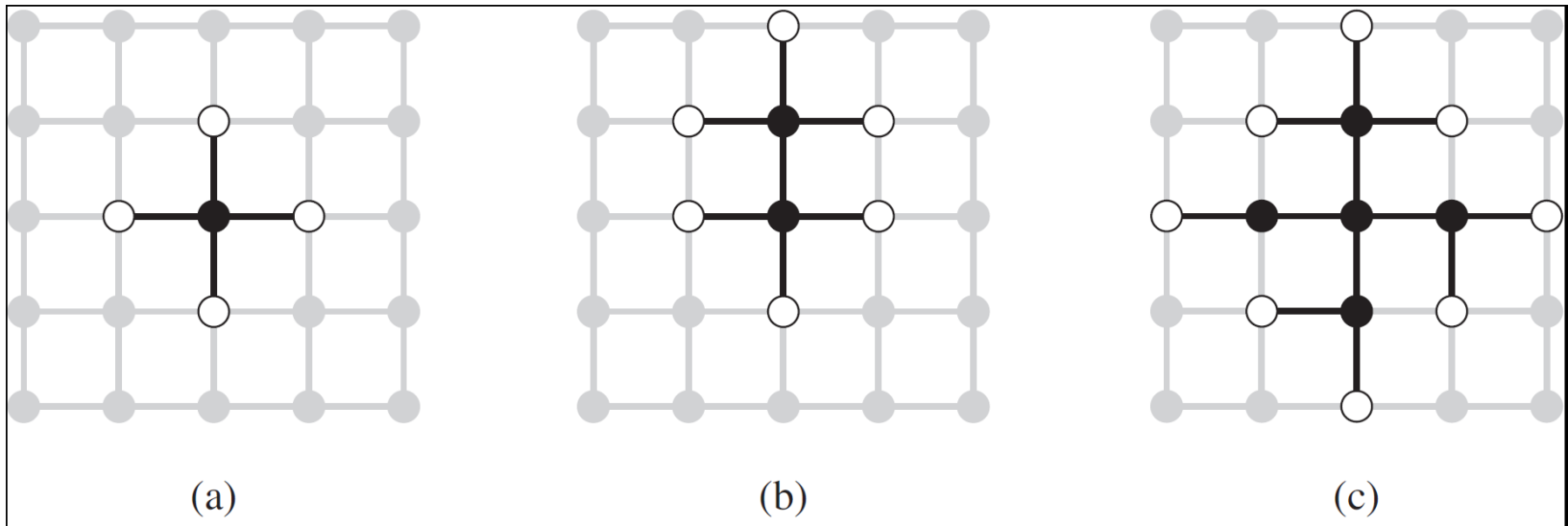
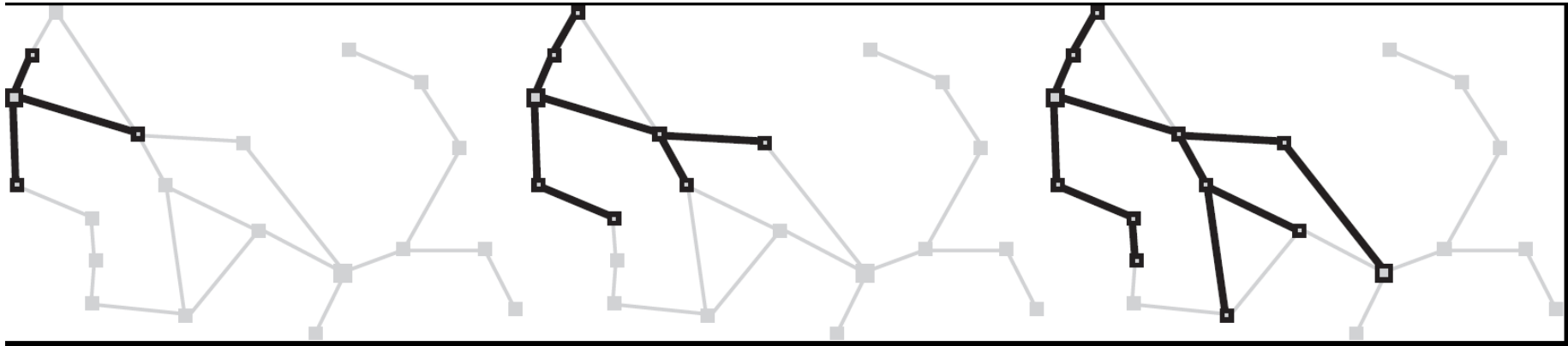
# Tree-Search vs Graph-Search

- **Tree-search(problem)**, returns a solution or failure
- Frontier  $\leftarrow$  initial state
- Loop do
  - If frontier is empty return failure
  - Choose a leaf node and remove from frontier
  - If the node is a goal, return the corresponding solution
  - Expand the chosen node, adding its children to the frontier
  - -----
- **Graph-search(problem)**, returns a solution or failure
- Frontier  $\leftarrow$  initial state, *explored*  $\leftarrow$  empty
- Loop do
  - If frontier is empty return failure
  - Choose a leaf node and remove from frontier
  - If the node is a goal, **return** the corresponding solution.
  - *Add the node to the explored.*
  - Expand the chosen node, adding its children to the frontier, *only if not in frontier or explored set*

# Basic search scheme

- We have 3 kinds of states
  - explored (past) – **only graph search**
  - frontier (current)
  - unexplored (future) – implicitly given
- Initially frontier=start state
- Loop until found solution or exhausted state space
  - pick/remove first node from frontier using search strategy
    - priority queue – FIFO (BFS), LIFO (DFS), g (UCS), f (A\*), etc.
  - check if goal
  - add this node to explored – **only graph search**
  - expand this node, add children to frontier (**graph search : only those children whose state is not in explored/frontier list**)
  - Q: what if better path is found to a node already on explored list?

# Graph-Search

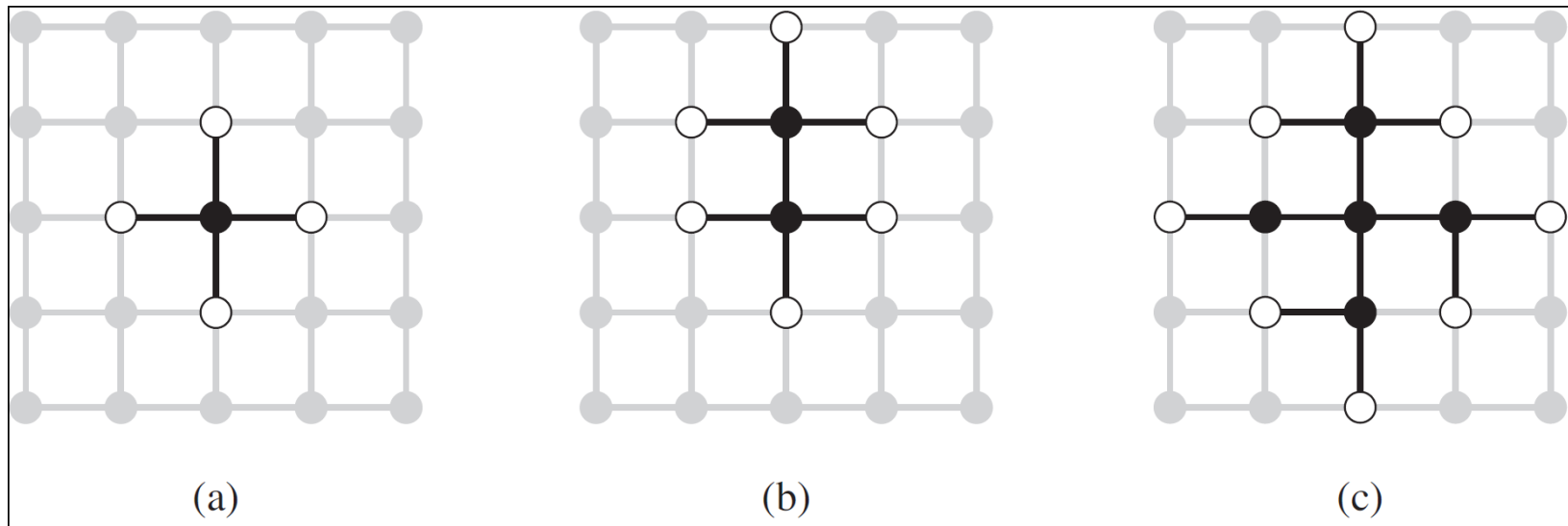


# Tree-Search vs. Graph-Search

- Example : Assemble 5 objects {**a**, **b**, **c**, **d**, **e**}
- A state is a bit-vector (length 5), 1=object in assembly
- 11010 = **a**, **b**, **d** in assembly, **c**, **e** not
- State space
  - number of states  $2^5 = 32$
  - number of edges  $(2^5) \cdot 5 \cdot \frac{1}{2} = 80$
- Tree-search space
  - number of nodes  $5! = 120$
- State can be reached in multiple ways
  - 11010 can be reached **a+b+d** or **a+d+b** etc.
- Graph-search :
  - three kinds of nodes : **unexplored**, **frontier**, **explored**
  - before adding a node, check if a state is in **frontier** or **explored** set

# Tree-Search vs. Graph-Search

- Route finding on rectangular grid (e.g. computer games)
  - Tree search  $O(4^d)$
  - Graph search  $O(d^2)$



# Why Search Can be Difficult

- **At the start of the search, the search algorithm does not know**
  - the size of the tree
  - the shape of the tree
  - the depth of the goal states
- **How big can a search tree be?**
  - say there is a constant branching factor  $b$
  - and one goal exists at depth  $d$
  - search tree which includes a goal can have  
 **$b^d$  different branches in the tree (worst case)**
- **Examples:**
  - $b = 2, d = 10: \quad b^d = 2^{10} = 1024$
  - $b = 10, d = 10: \quad b^d = 10^{10} = 10,000,000,000$

# Searching the Search Space

- Uninformed (Blind) search : don't know if a state is "good"
  - Breadth-first
  - Uniform-Cost first
  - Depth-first
  - Iterative deepening depth-first
  - Bidirectional
  - Depth-First Branch and Bound
- Informed Heuristic search : have evaluation fn for states
  - Greedy search, hill climbing, Heuristics
- Important concepts:
  - Completeness : does it always find a solution if one exists ?
  - Time complexity (b, d, m)
  - Space complexity (b, d, m)
  - Quality of solution : optimality = does it always find best solution?



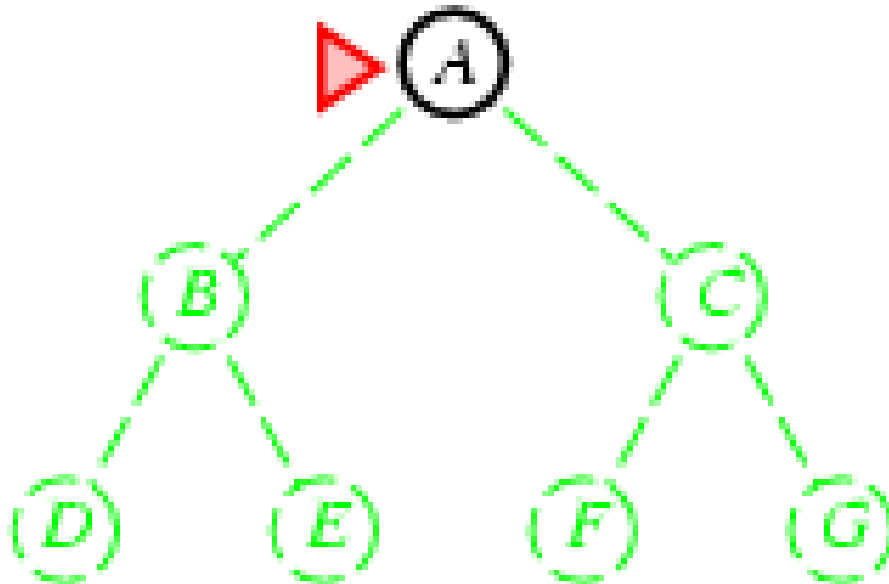
# Search strategies

- A search **strategy** is defined by picking the **order of node expansion**
- Strategies are evaluated along the following dimensions:
  - **completeness**: does it always find a solution if one exists?
  - **time complexity**: number of nodes generated
  - **space complexity**: maximum number of nodes in memory
  - **optimality**: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b*: maximum branching factor of the search tree
  - *d*: depth of the least-cost solution
  - *m*: maximum depth of the state space (may be  $\infty$ )

# Breadth-First Search

- Expand shallowest unexpanded node
- Frontier: nodes waiting in a queue to be explored, also called **OPEN**
- **Implementation:**
  - *frontier* is a first-in-first-out (FIFO) queue, i.e., new successors go at end of the queue.

Is A a goal state?

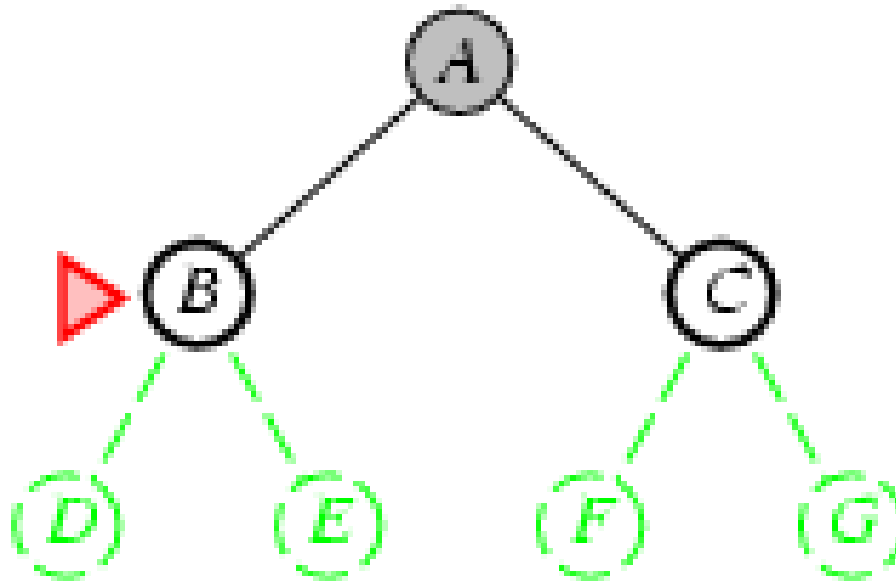


# Breadth-First Search

- Expand shallowest unexpanded node
- **Implementation:**
  - *frontier* is a FIFO queue, i.e., new successors go at end

Expand:  
frontier = [B,C]

Is B a goal state?

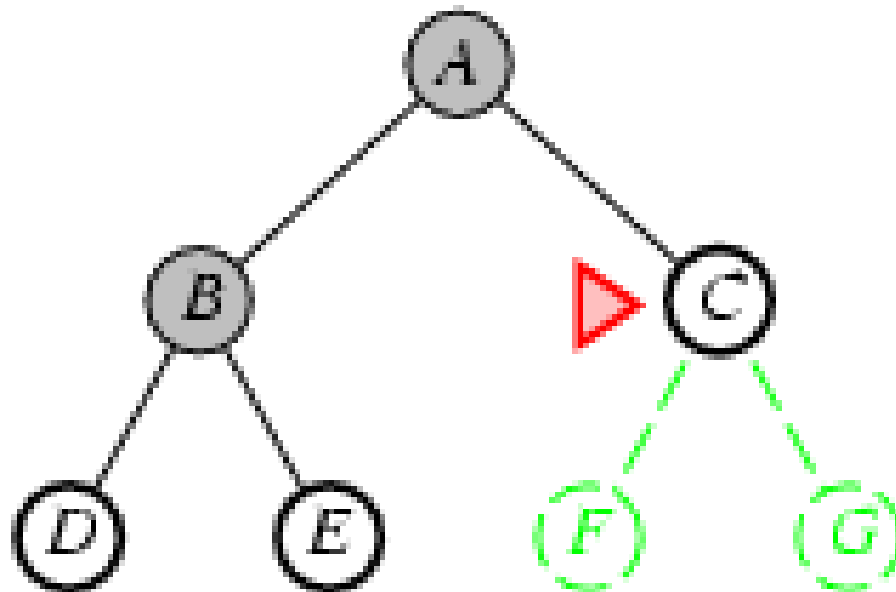


# Breadth-First Search

- Expand shallowest unexpanded node
- **Implementation:**
  - *frontier* is a FIFO queue, i.e., new successors go at end

Expand:  
frontier=[C,D,E]

Is C a goal state?

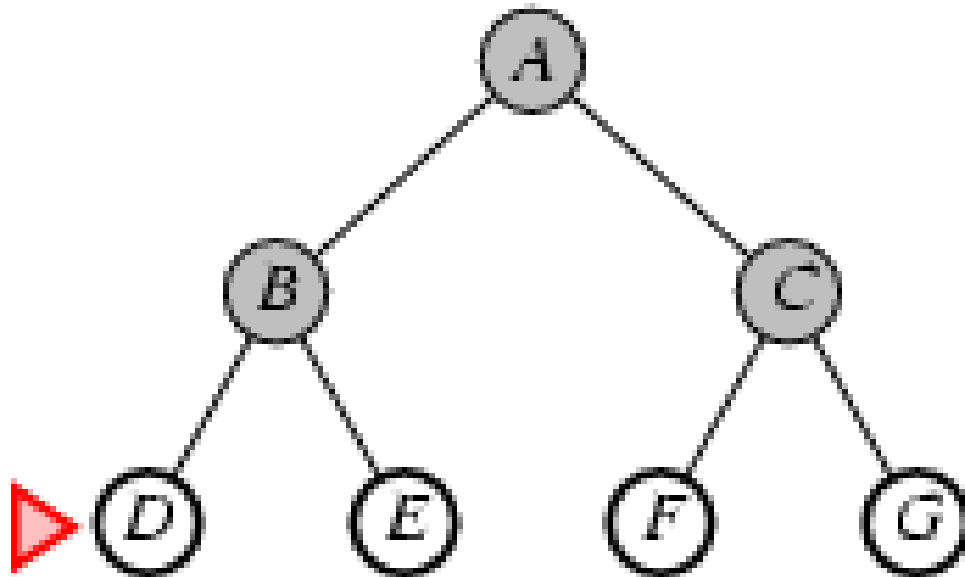


# Breadth-First Search

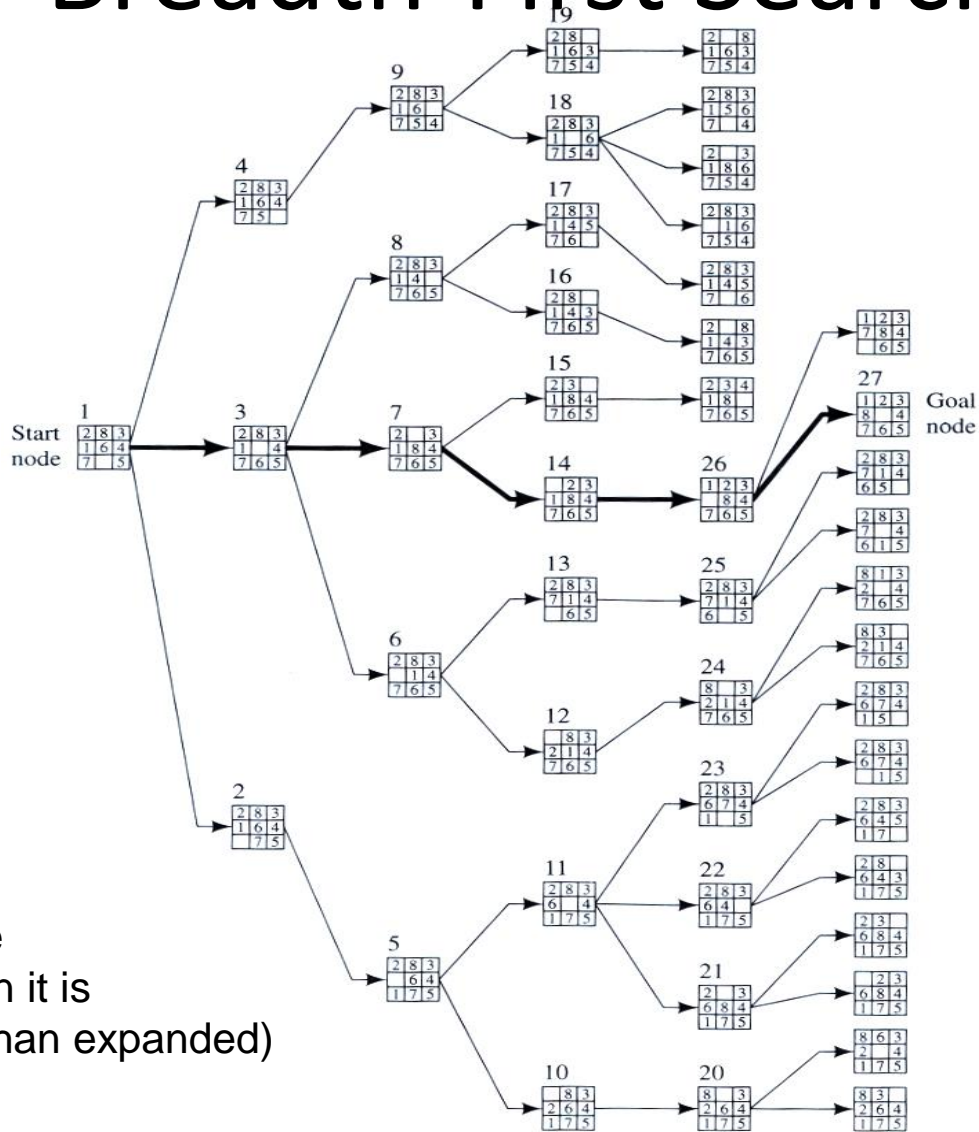
- Expand shallowest unexpanded node
- **Implementation:**
  - *frontier* is a FIFO queue, i.e., new successors go at end

Expand:  
frontier=[D,E,F,G]

Is D a goal state?



# Breadth-First Search



Actually, in BFS we can check if a node is a goal node when it is generated (rather than expanded)

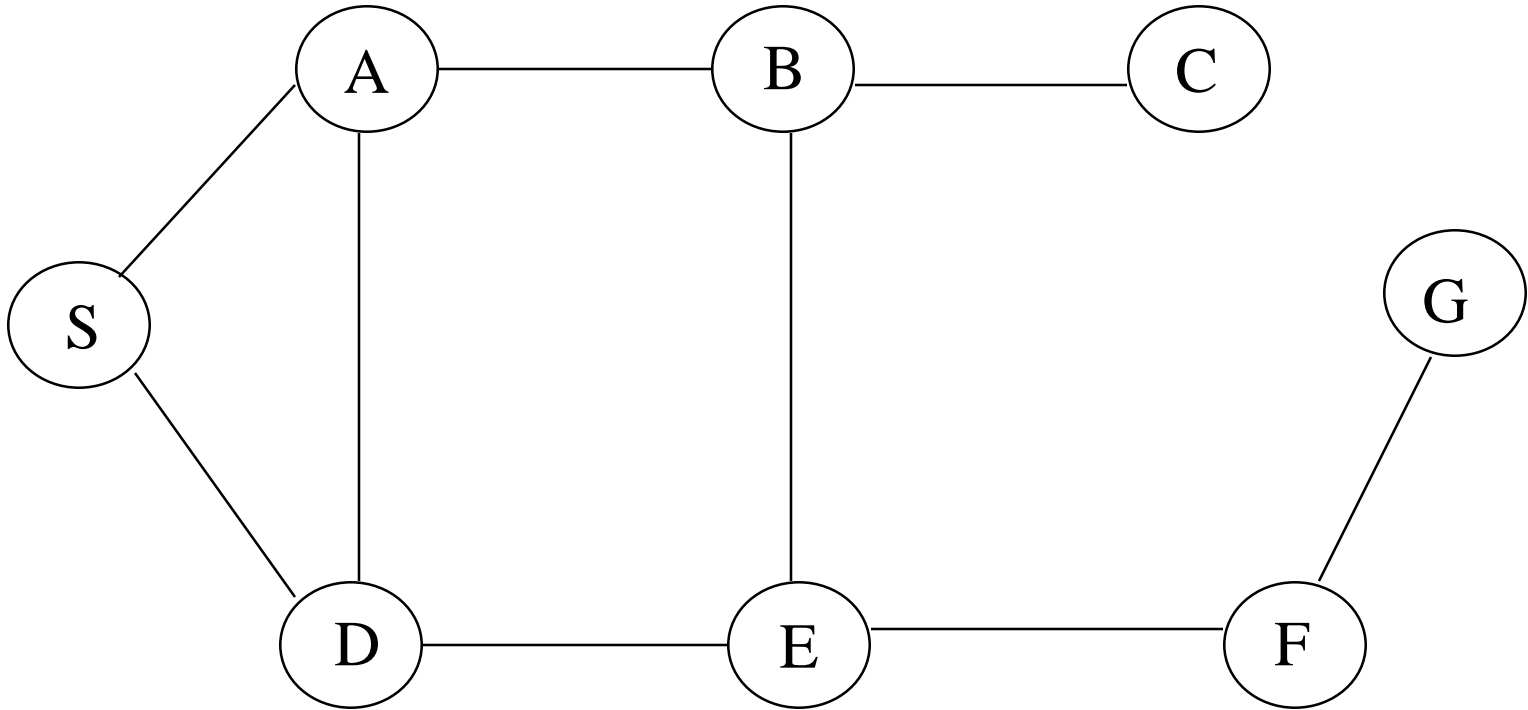
# Breadth-First-Search (\*)

OPEN = frontier, CLOSED = explored

- 1. Put the start node  $s$  on OPEN
- 2. If OPEN is empty exit with failure.
- 3. Remove the first node  $n$  from OPEN and place it on CLOSED.
- 4. Expand  $n$ , generating all its successors.
  - If child is not in CLOSED or OPEN, then
  - If child is not a goal, then put them at the end of OPEN in some order.
- 5. If  $n$  is a goal node, exit successfully with the solution obtained by tracing back pointers from  $n$  to  $s$ .
- Go to step 2.

\* This is graph-search

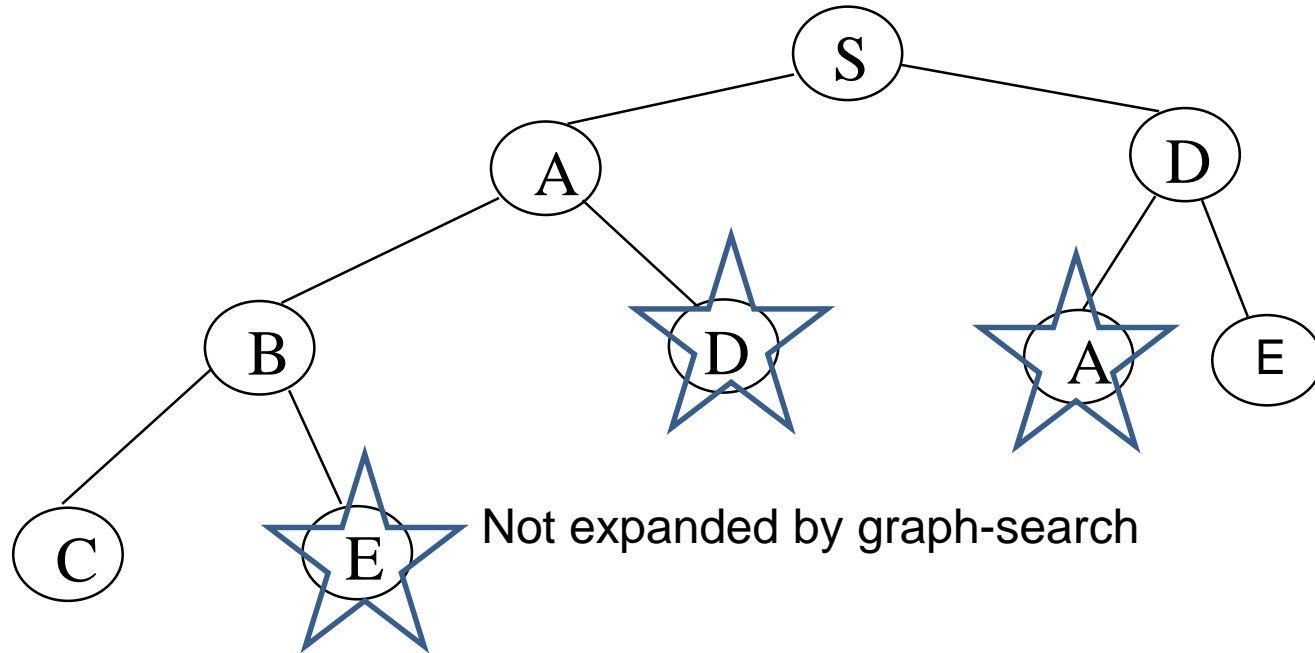
# Example: Map Navigation



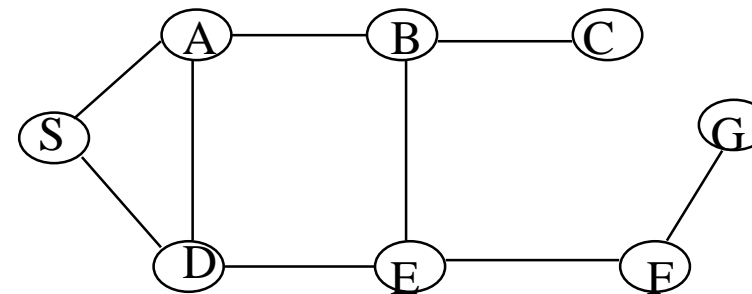
S = start, G = goal, other nodes = intermediate states, links = legal transitions



# Breadth-First Search Graph



Note: this is the search tree at some particular point in the search.

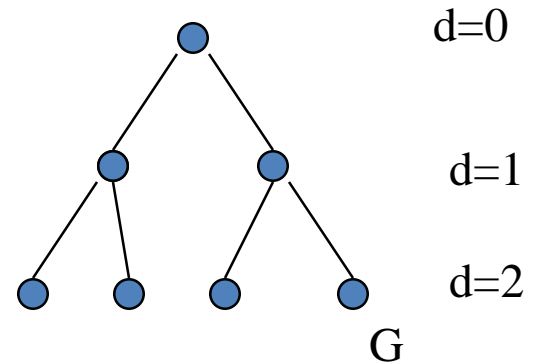


# Complexity of Breadth-First Search

- **Time Complexity**

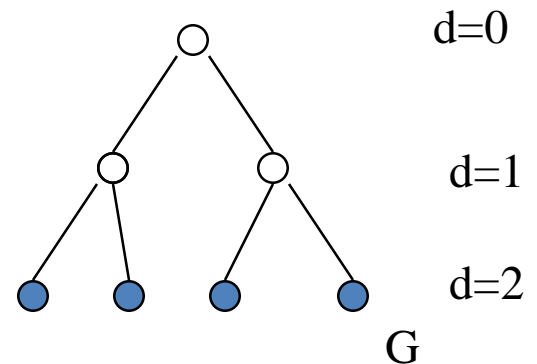
- assume (worst case) that there is 1 goal leaf at the RHS
- so BFS will expand all nodes

$$= 1 + b + b^2 + \dots + b^d$$
$$= \mathbf{O(b^d)}$$



- **Space Complexity**

- how many nodes can be in the queue (worst-case)?
- at depth  $d$  there are  $b^d$  unexpanded nodes in the Q =  $\mathbf{O(b^d)}$



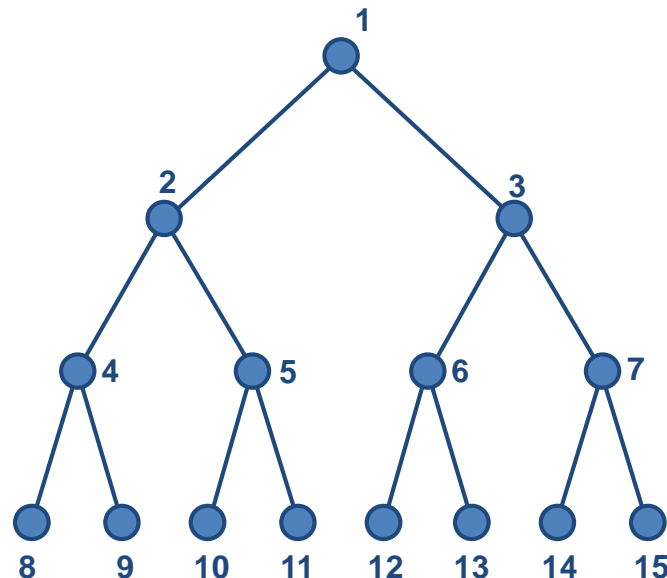
# Examples of Time and Memory Requirements for Breadth-First Search

| Depth of Solution | Nodes Expanded | Time          | Memory        |
|-------------------|----------------|---------------|---------------|
| 0                 | 1              | 1 millisecond | 100 bytes     |
| 2                 | 111            | 0.1 seconds   | 11 kbytes     |
| 4                 | 11,111         | 11 seconds    | 1 megabyte    |
| 8                 | $10^8$         | 31 hours      | 11 giabytes   |
| 12                | $10^{12}$      | 35 years      | 111 terabytes |

Assuming  $b=10$ , 1000 nodes/sec, 100 bytes/node

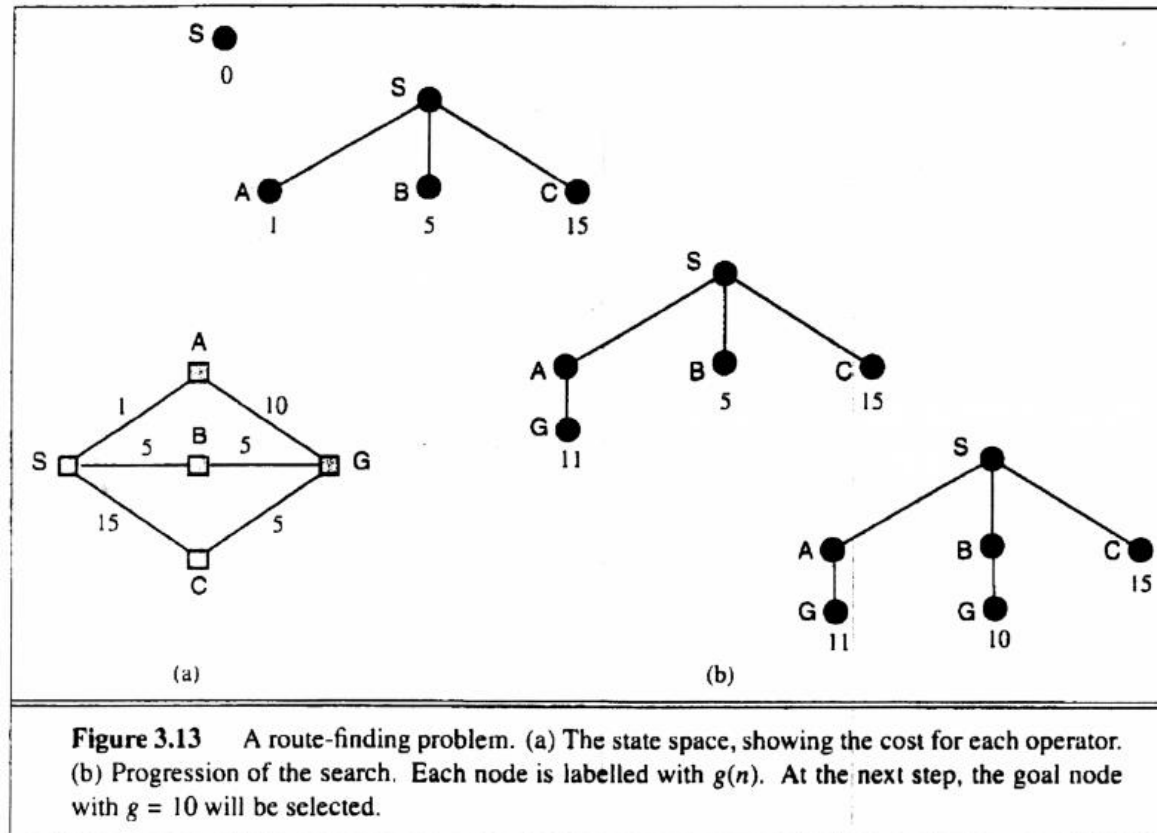
# Breadth-First Search (BFS) Properties

- Solution Length: optimal
- Expand each node once (can check for duplicates, performs graph-search)
- Search Time:  $O(b^d)$
- Memory Required:  $O(b^d)$
- Drawback: requires exponential space



# Uniform Cost Search

- Use priority queue to implement frontier
- Expand lowest-cost OPEN node ( $g(n)$ )
- In BFS  $g(n) = \text{depth}(n)$



## ○ Requirement

- $g(\text{successor}(n)) \geq g(n)$

# Uniform Cost Search

- Guaranteed to find optimal solution (as long as all steps have  $>0$  cost)
  - When a node is selected for expansion, a shortest path to it has been found
- UCS expands in the order of optimal path cost

# Uniform cost search

1. Put the start node  $s$  on OPEN
2. If OPEN is empty exit with failure.
3. Remove the first node  $n$  from OPEN and place it on CLOSED.
4. If  $n$  is a goal node, exit successfully with the solution obtained by tracing back pointers from  $n$  to  $s$ .
5. Otherwise, expand  $n$ , generating all its successors attach to them pointers back to  $n$ , and put them in OPEN *in order of shortest cost*
6. Go to step 2.

## DFS Branch and Bound

***At step 4: compute the cost of the solution found and update the upper bound  $U$ .***

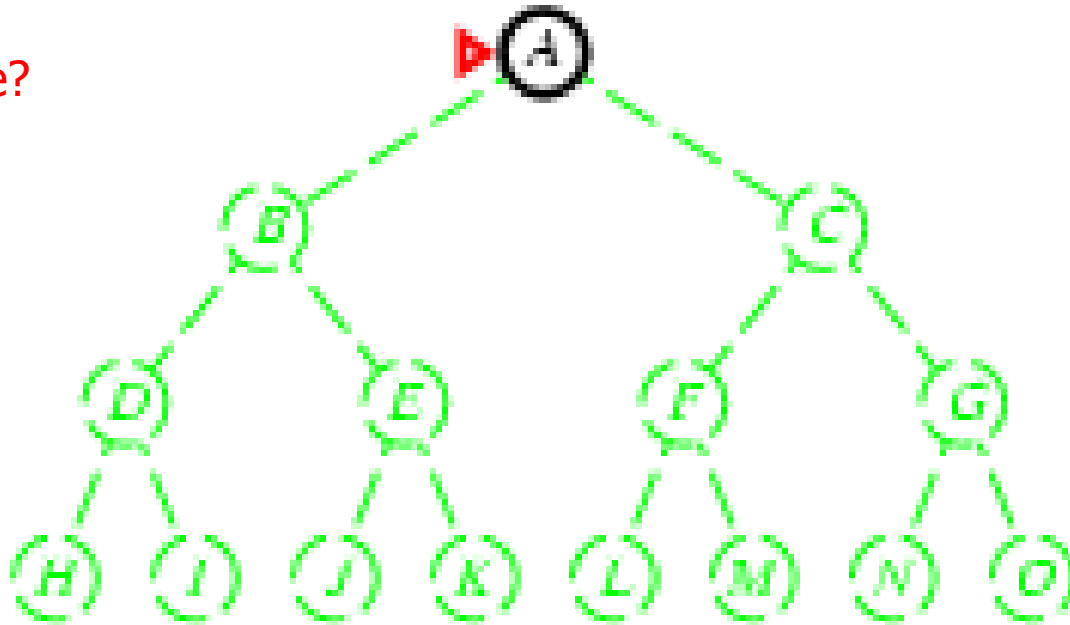
***At step 5: expand  $n$ , generating all its successors attach to them pointers back to  $n$ , and put on top of OPEN.***

***Compute cost of partial path to node and prune if larger than  $U$ .***

# Depth-First Search

- Expand *deepest* unexpanded node
- **Implementation:**
  - *frontier* = Last In First Out (LIFO) queue, i.e., put successors at front

Is A a goal state?



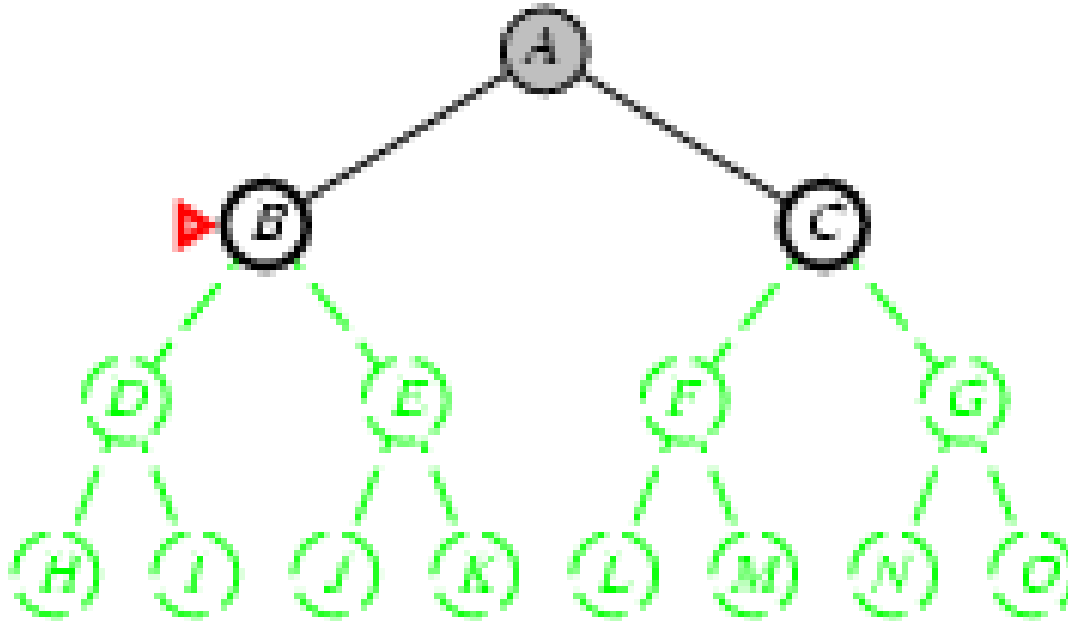


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[B,C]

Is B a goal state?

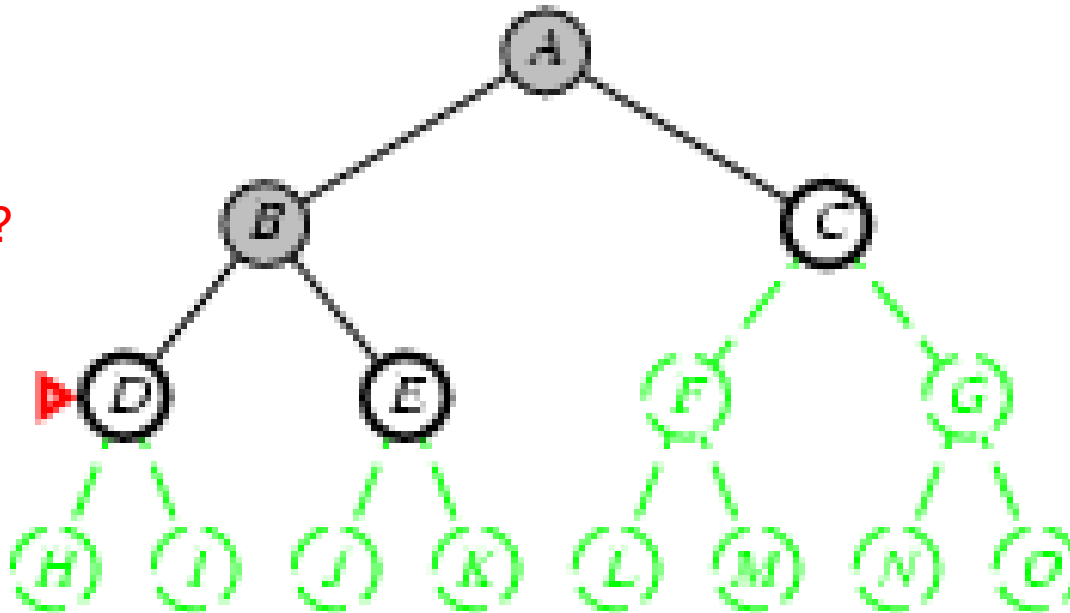


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[D,E,C]

Is D = goal state?

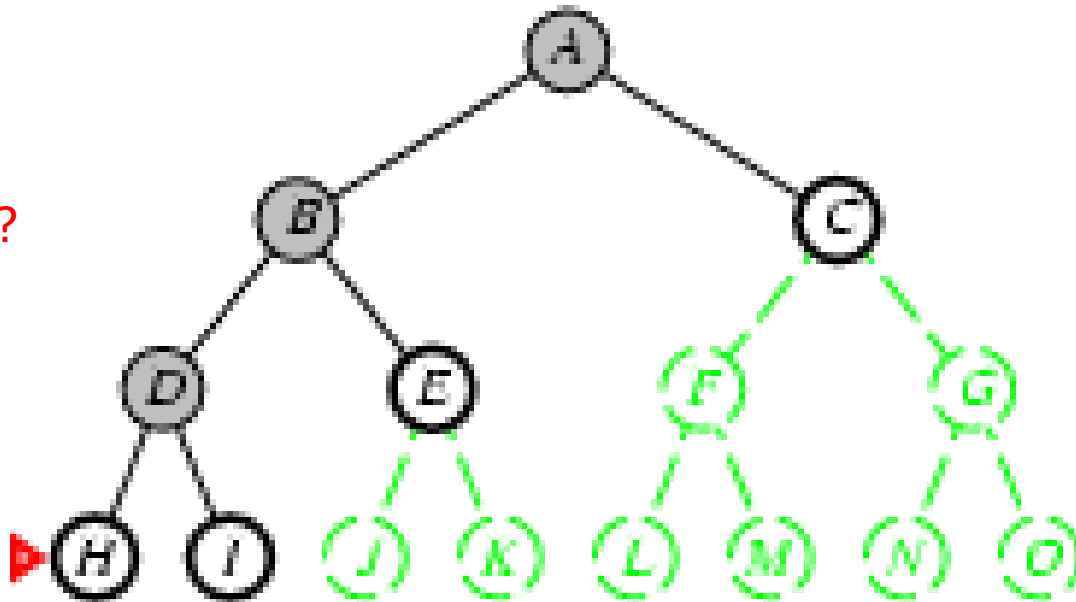


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[H,I,E,C]

Is H = goal state?

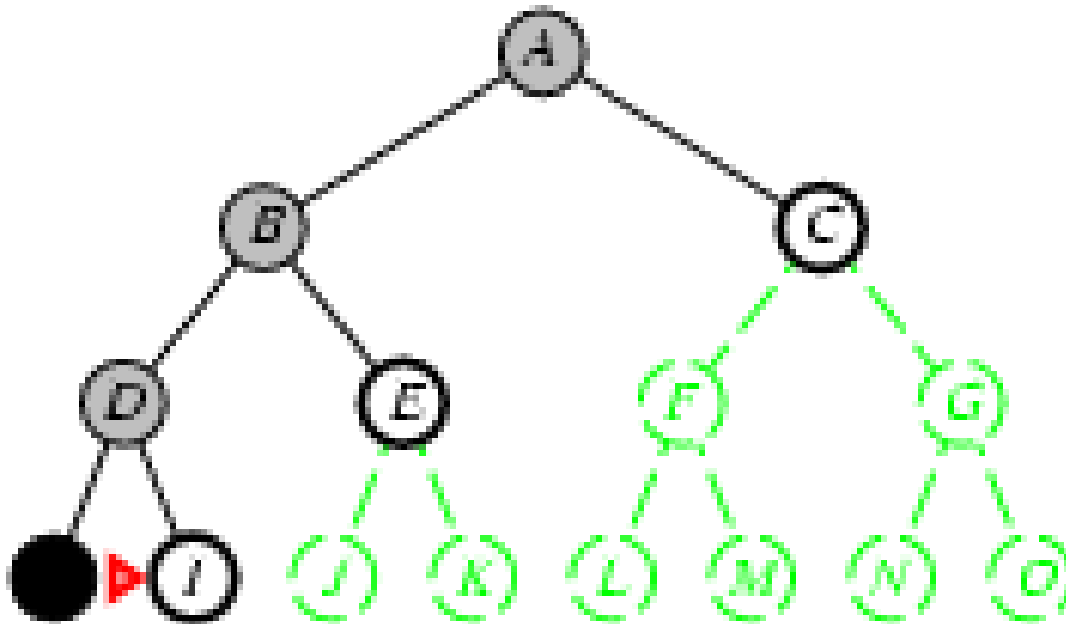


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[I,E,C]

Is I = goal state?

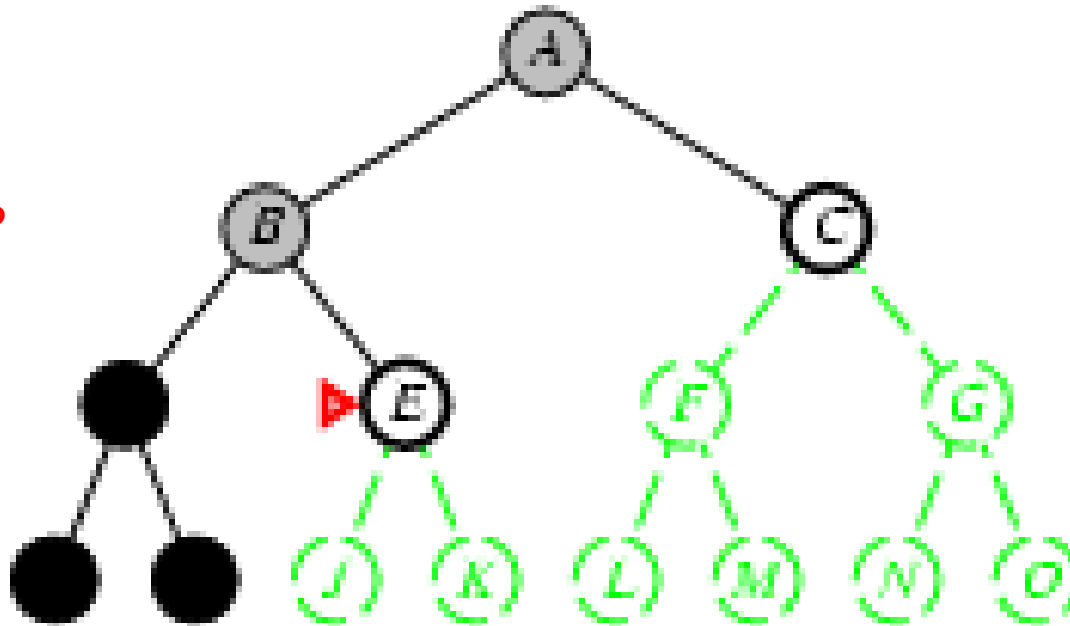


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[E,C]

Is E = goal state?

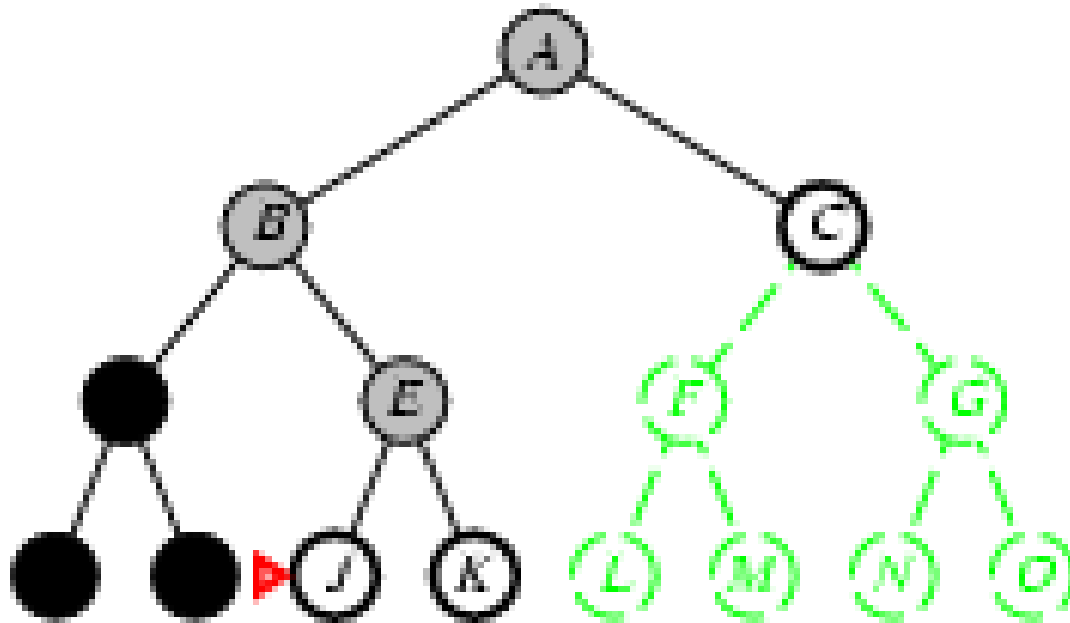


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[J,K,C]

Is J = goal state?

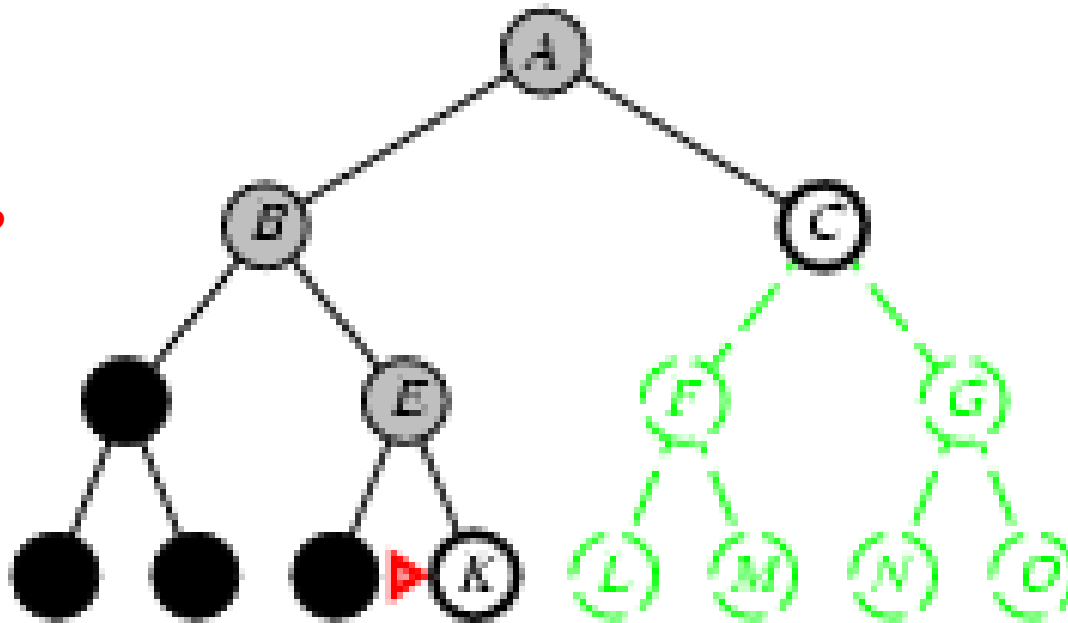


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[K,C]

Is K = goal state?

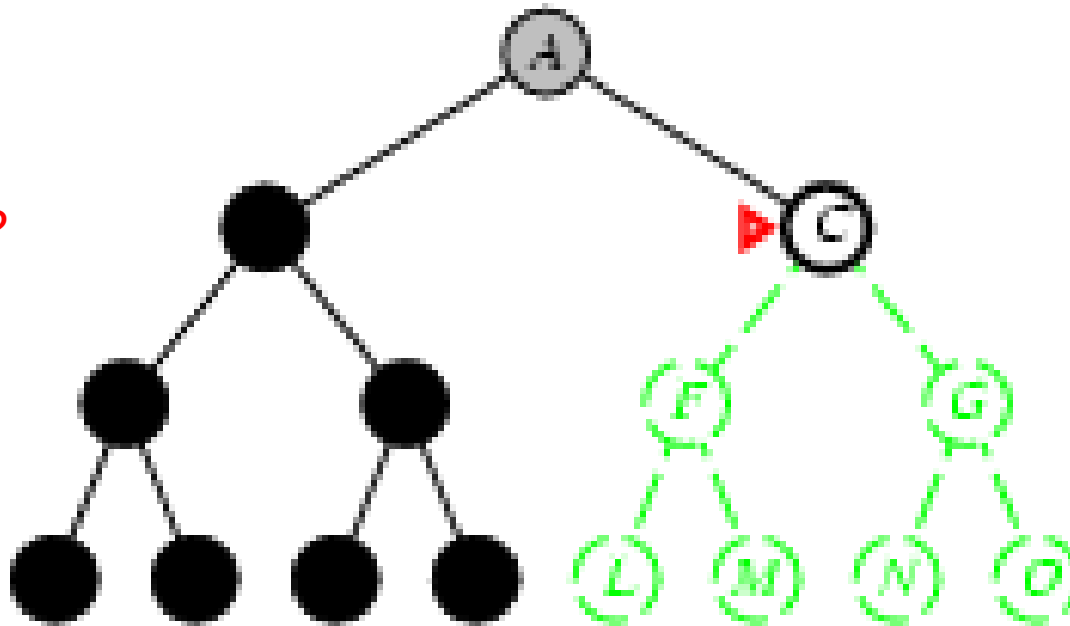


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[C]

Is C = goal state?



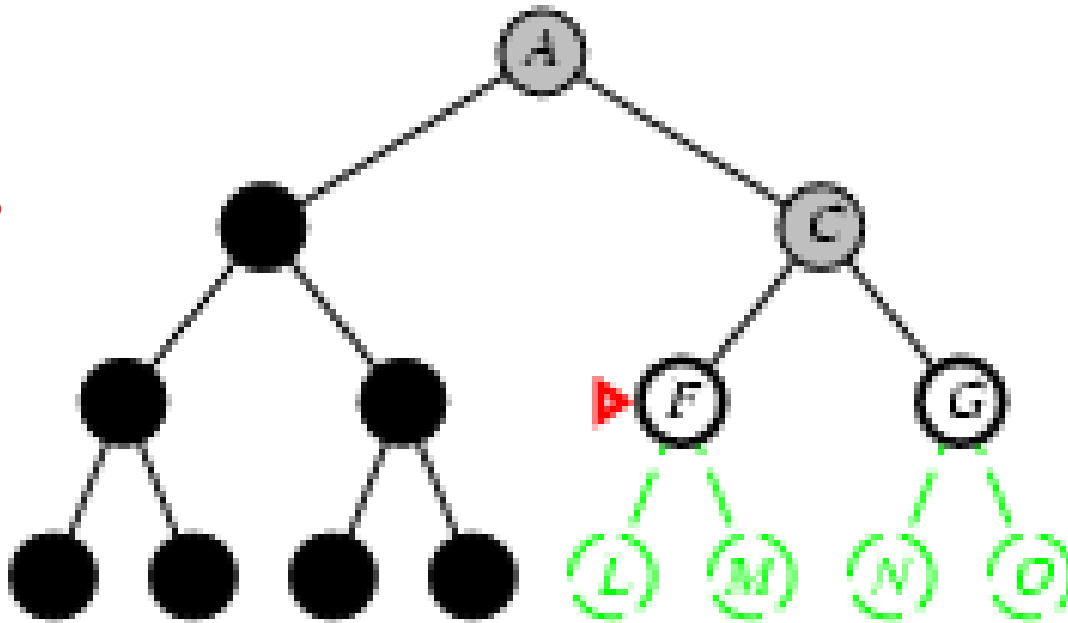


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[F,G]

Is F = goal state?

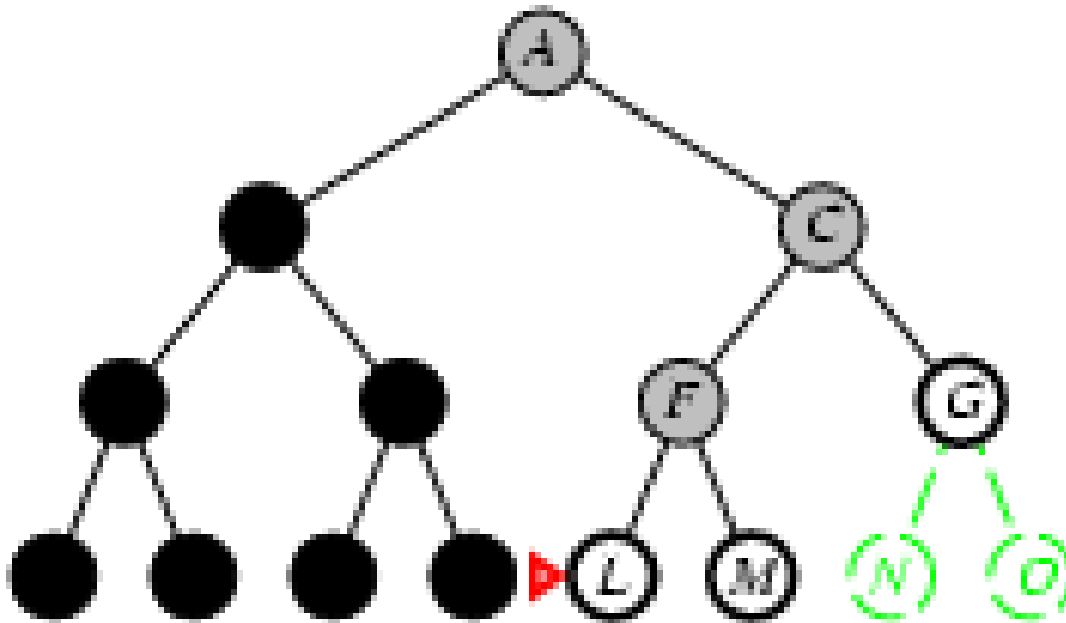


# Depth-first search

- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[L,M,G]

Is L = goal state?

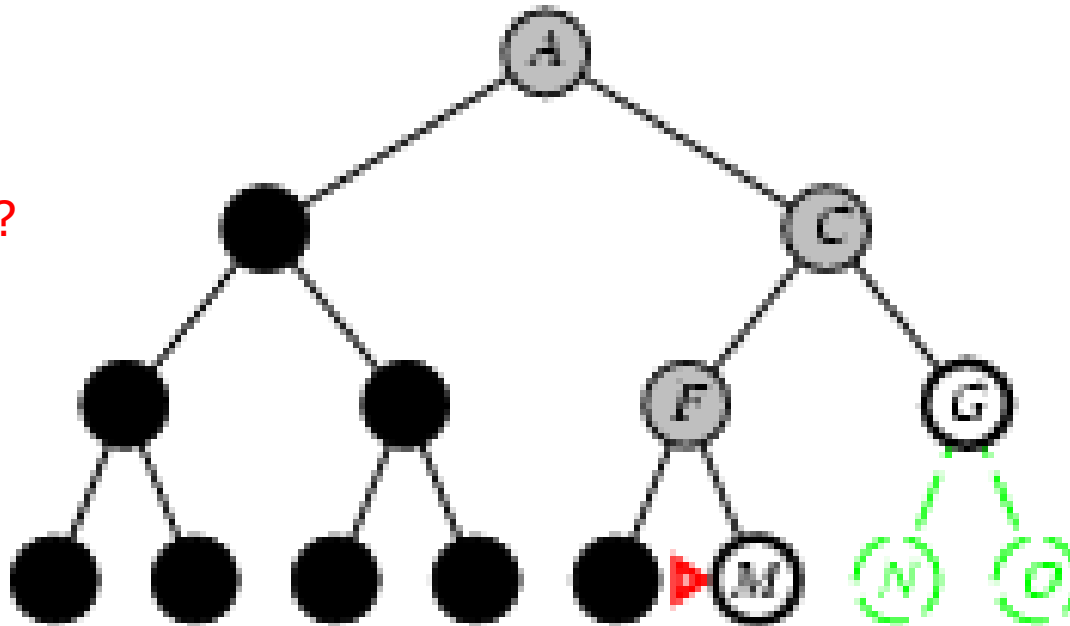


# Depth-first search

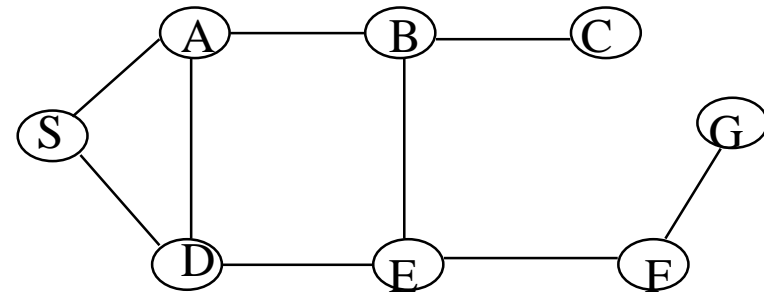
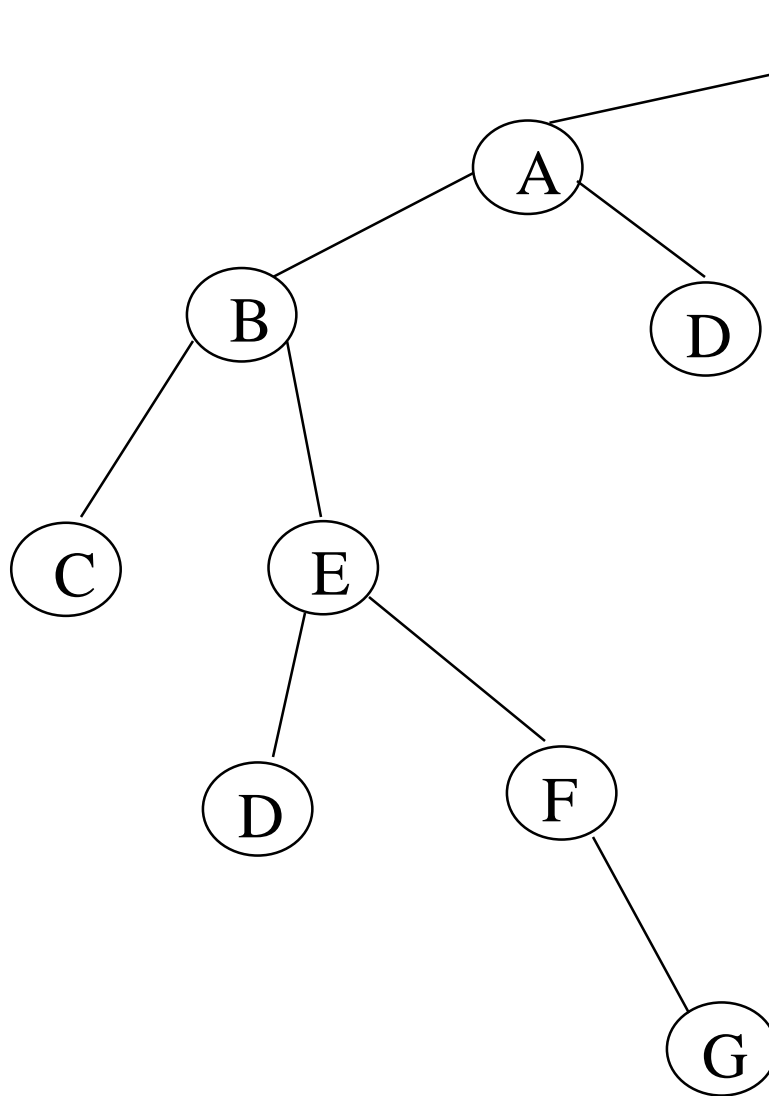
- Expand deepest unexpanded node
- **Implementation:**
  - *frontier* = LIFO queue, i.e., put successors at front

queue=[M,G]

Is M = goal state?

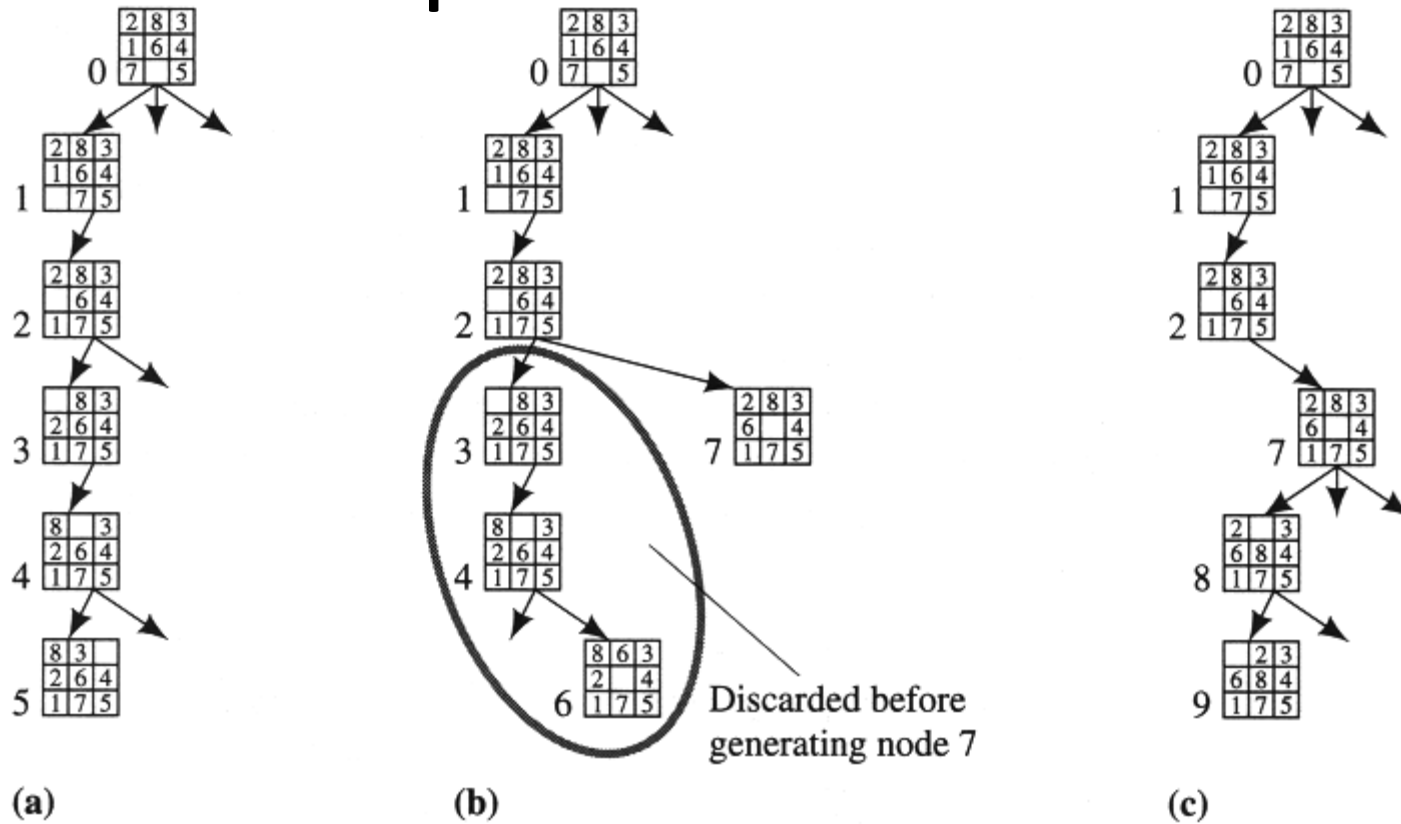


# Depth-First Search (DFS)

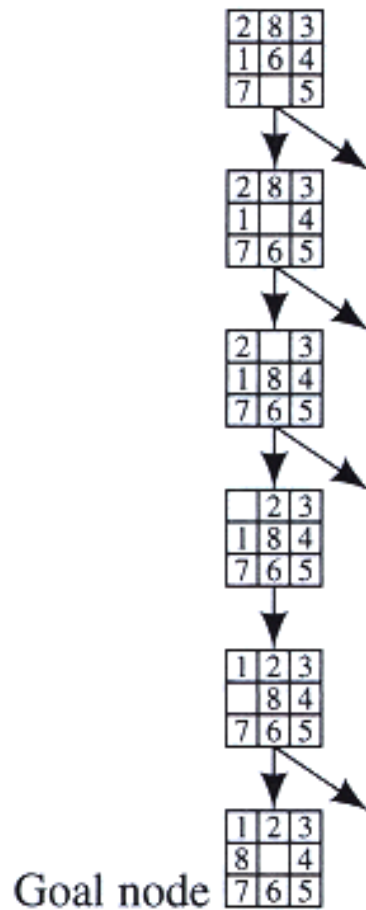


Here, (if tree-search) then to avoid infinite depth (in case of finite state-space graph) assume we don't expand any child node which appears already in the path from the root S to the parent. (Again, one could use other strategies)

# Depth-First Search



Generation of the First Few Nodes in a Depth-First Search




---

The Graph When the Goal Is Reached in Depth-First Search

# Depth-First-Search (\*)

1. Put the start node  $s$  on OPEN
2. If OPEN is empty exit with failure.
3. Remove the first node  $n$  from OPEN.
4. If  $n$  is a goal node, exit successfully with the solution obtained by tracing back pointers from  $n$  to  $s$ .
5. Otherwise, expand  $n$ , generating all its successors (check for self-loops) attach to them pointers back to  $n$ , and put them at the top of OPEN *in some order*.
6. Go to step 2.

\*search the tree search-space (but avoid self-loops)

\*\* the default assumption is that DFS searches the underlying search-tree

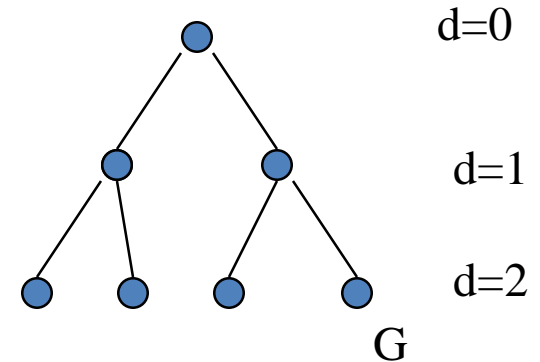
# Complexity of Depth-First Search?

- Time Complexity

- assume  $d$  is deepest path in the search space
- assume (worst case) that there is 1 goal leaf at the RHS
- so DFS will expand all nodes

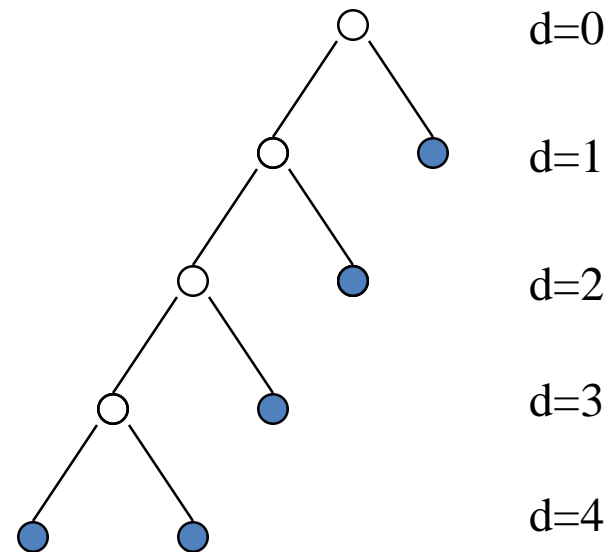
$$= 1 + b + b^2 + \dots + b^d$$

$$= O(b^d)$$



- Space Complexity (for tree-search)

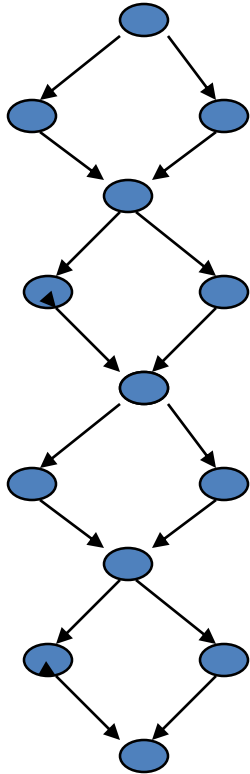
- how many nodes can be in the queue (worst-case)?
- **$O(bd)$  if deepest node at depth  $d$**





# Example, Diamond Networks

## graph-search vs tree-search (BFS vs DFS)



- Graph-search & BFS
- Tree-search & DFS

# Depth-First tree-search Properties

- Non-optimal solution path
- Incomplete unless there is a depth bound
- (we will assume depth-limited DF-search)
- Re-expansion of nodes (when the state-space is a graph)
- Exponential time
- Linear space (for tree-search)

# Comparing DFS and BFS

- BFS optimal, DFS is not
- Time Complexity worse-case is the same, but
  - In the worst-case BFS is always better than DFS
  - Sometimes, on the average DFS is better if:
    - many goals, no loops and no infinite paths
- BFS is much worse memory-wise
  - DFS can be linear space
  - BFS may store the whole search space.
- In general
  - BFS is better if goal is not deep, if long paths, if many loops, if small search space
  - DFS is better if many goals, not many loops
  - DFS is much better in terms of memory

# Iterative-Deepening Search (DFS)

- Every iteration is a DFS with a depth cutoff.

## Iterative deepening (ID)

1.  $i = 1$
2. While no solution, do
3. DFS from initial state  $S_0$  with cutoff  $i$
4. If found goal, stop and return solution, else, increment cutoff

## Comments:

- IDS implements BFS with DFS
- Only one path in memory
- BFS at step  $i$  may need to keep  $2^i$  nodes in OPEN

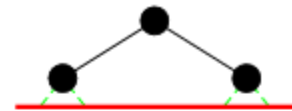
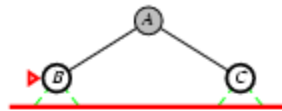
# Iterative deepening search $L=0$

Limit = 0



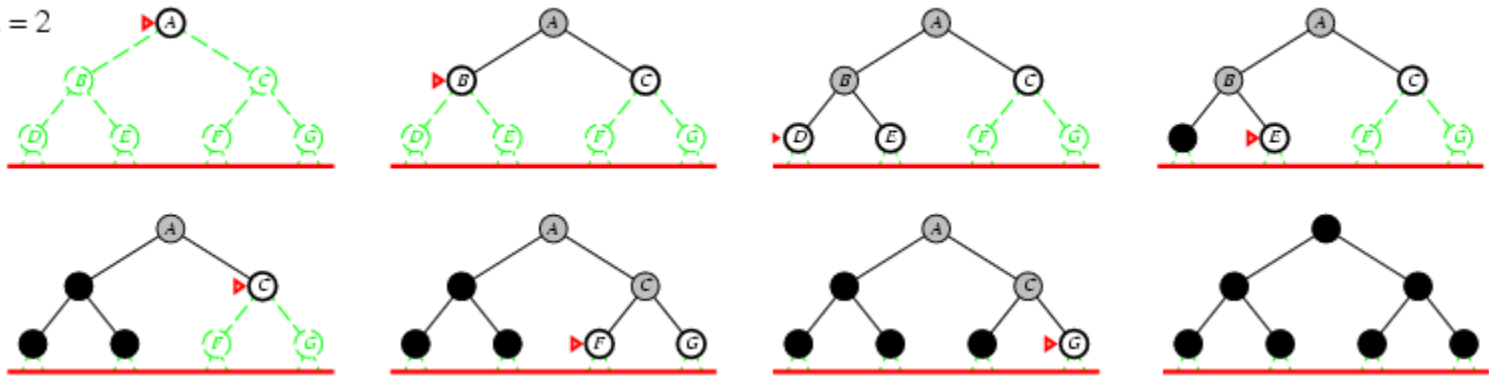
# Iterative deepening search $L=1$

Limit = 1



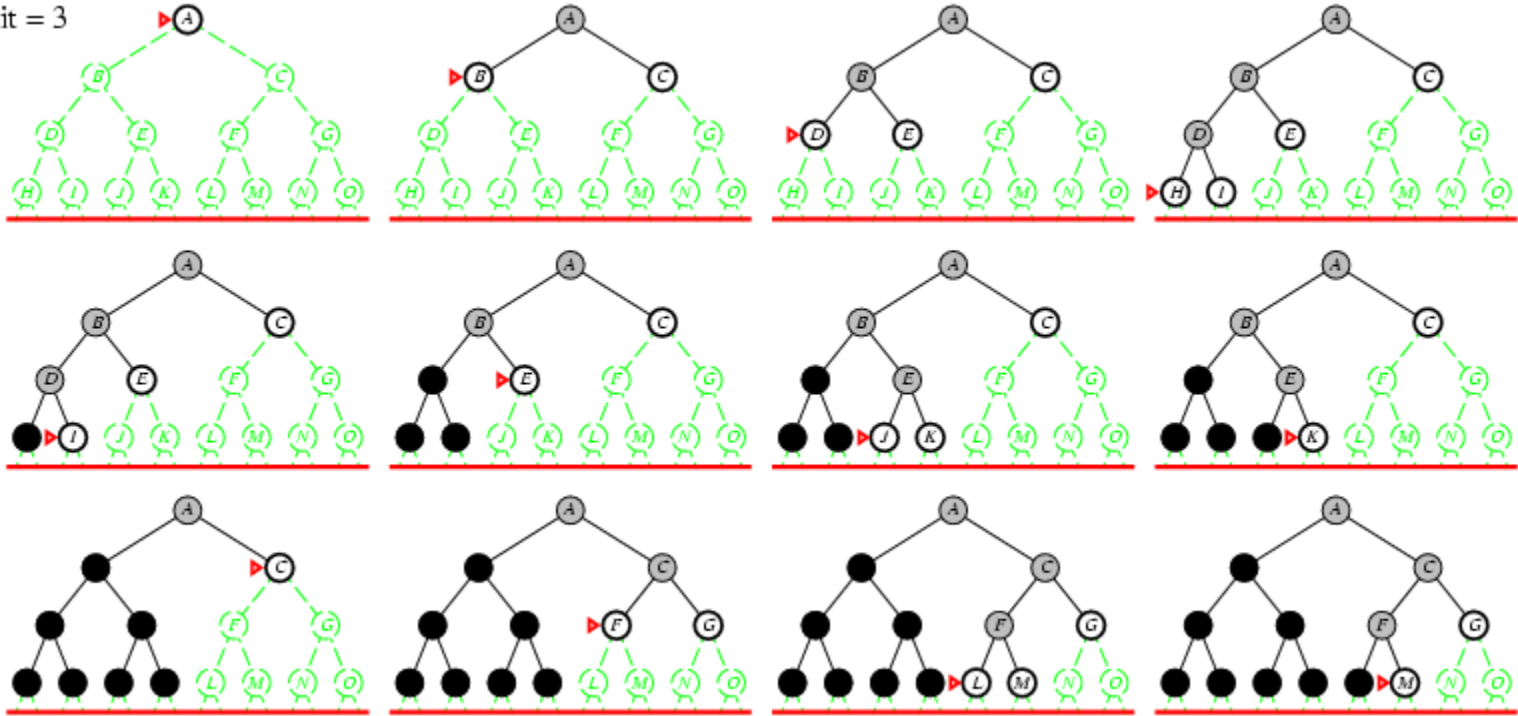
# Iterative deepening search $L=2$

Limit = 2



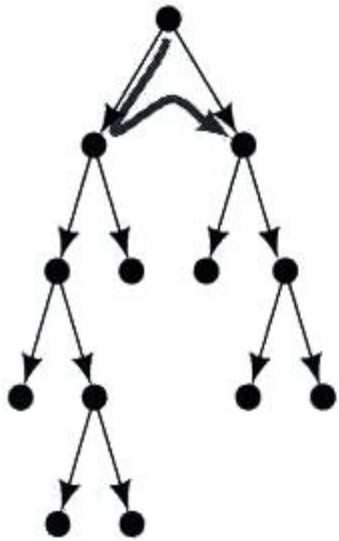
# Iterative Deepening Search $L=3$

Limit = 3

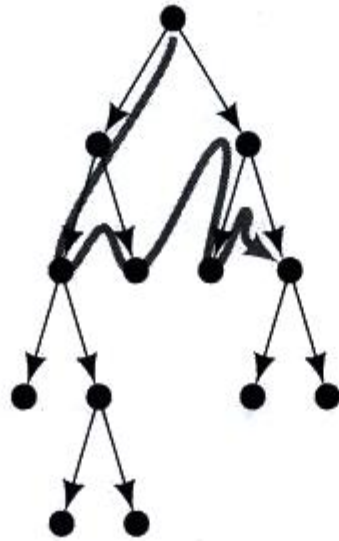




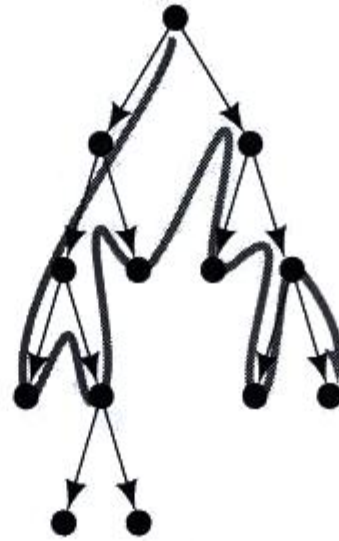
# Iterative deepening search



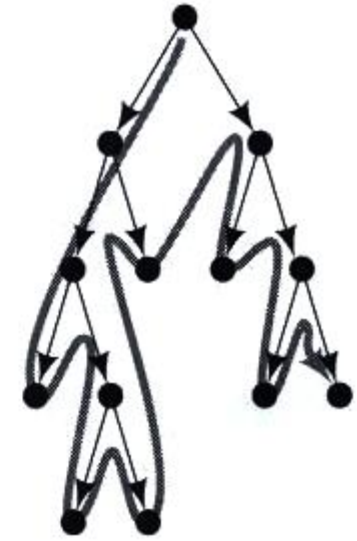
Depth bound = 1



Depth bound = 2



Depth bound = 3



Depth bound = 4

---

Stages in Iterative-Deepening Search

# Iterative Deepening (DFS)

- Time: 
$$T(n) = \sum_{j=1}^n \frac{b^{j+1} - 1}{b - 1} = \frac{b^{n+2}}{(b-1)^2} = O(b^n)$$

- BFS time is  $O(b^n)$ ,  $b$  is the branching degree
- IDS is asymptotically like BFS,
- For  $b=10$   $d=5$   $d=\text{cut-off}$
- DFS =  $1+10+100,\dots,=111,111$
- IDS = 123,456
- Ratio is  $\frac{b}{b-1}$

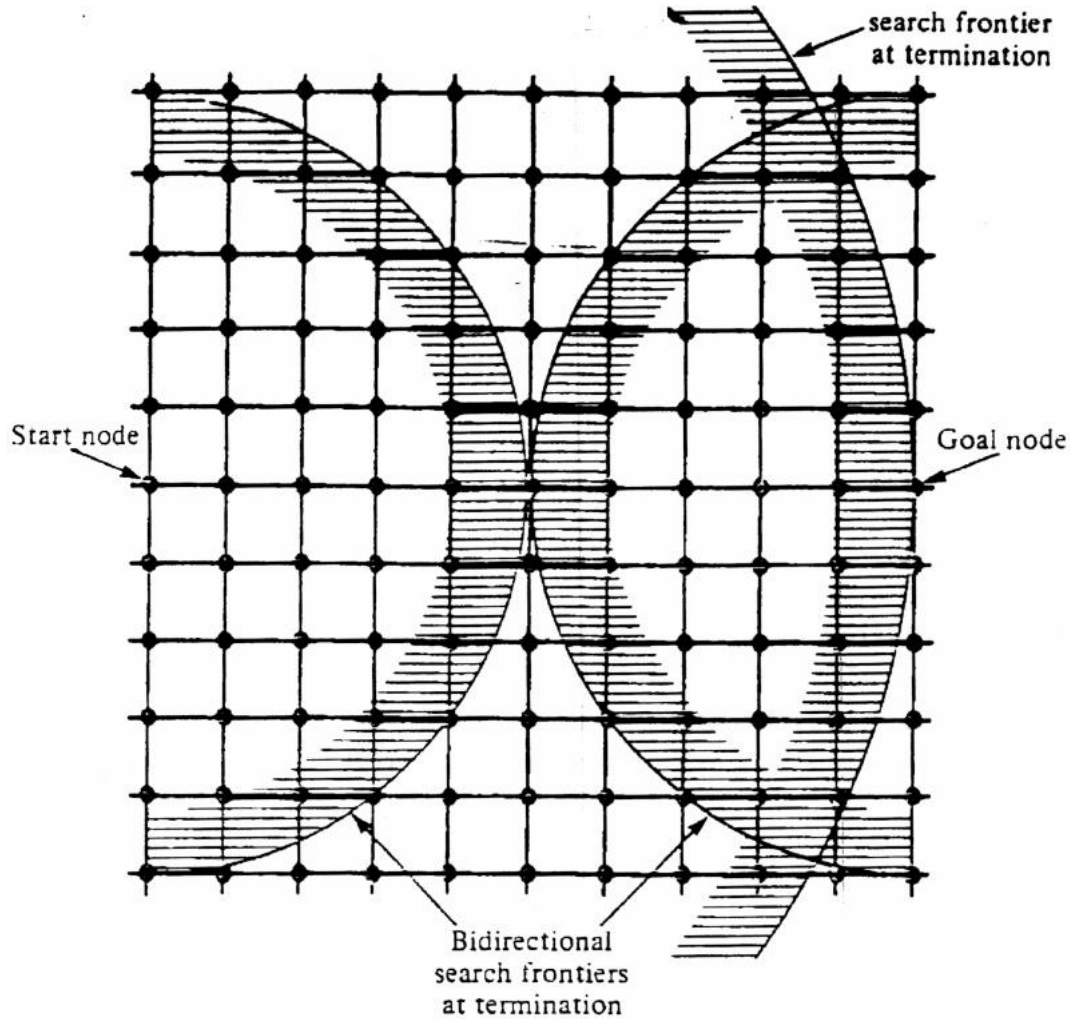
# Summary on IDS

- A useful practical method
  - combines
    - guarantee of finding an optimal solution if one exists (as in BFS)
    - space efficiency,  $O(bd)$  of DFS
    - But still has problems with loops like DFS

# Bidirectional Search

- Idea
  - simultaneously search forward from S and backwards from G
  - stop when both “meet in the middle”
  - need to keep track of the intersection of 2 open sets of nodes
- What does searching backwards from G mean
  - need a way to specify the predecessors of G
    - this can be difficult,
    - e.g., predecessors of checkmate in chess?
  - what if there are multiple goal states?
  - what if there is only a goal test, no explicit list?
- Complexity
  - time complexity is best:  $O(2 b^{(d/2)}) = O(b^{(d/2)})$
  - memory complexity is the same

# Bi-Directional Search



# Comparison of Algorithms

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited      | Iterative Deepening | Bidirectional (if applicable) |
|-----------|---------------|--------------|-------------|--------------------|---------------------|-------------------------------|
| Time      | $b^d$         | $b^d$        | $b^m$       | $b^l$              | $b^d$               | $b^{d/2}$                     |
| Space     | $b^d$         | $b^d$        | $bm$        | $bl$               | $bd$                | $b^{d/2}$                     |
| Optimal?  | Yes           | Yes          | No          | No                 | Yes                 | Yes                           |
| Complete? | Yes           | Yes          | No          | Yes, if $l \geq d$ | Yes                 | Yes                           |

**Figure 3.18** Evaluation of search strategies.  $b$  is the branching factor;  $d$  is the depth of solution;  $m$  is the maximum depth of the search tree;  $l$  is the depth limit.

# Summary

- A review of search
  - a search space consists of nodes and operators: it is a tree/graph
- There are various strategies for “uninformed search”
  - breadth-first
  - depth-first
  - iterative deepening
  - bidirectional search
  - Uniform cost search
  - Depth-first branch and bound
- Repeated states can lead to infinitely large search trees
  - we looked at methods for detecting repeated states
- All of the search techniques so far are “blind” in that they do not look at how far away the goal may be: next we will look at informed or heuristic search, which directly tries to minimize the distance to the goal.